



Investments into education—Doing as the parents did

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ABSTRACT

Empirical evidence suggests that parents with higher levels of education generally attach a higher importance to the education of their children. This implies an intergenerational chain transmitting the attitude towards the formation of human capital from one generation to the next. We incorporate this intergenerational chain into an OLG-model with endogenous human capital formation. In absence of any state intervention such an economy might be characterized by multiple steady states with low or high human capital levels. There are also steady states where the population is permanently divided into different groups with differing human capital and welfare levels. Compulsory schooling is needed to overcome steady states with low human capital and welfare levels. Tax financed education subsidies can lead to further pareto-improvements.

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1. Introduction

Human capital is one of the most important determinants of economic progress and welfare. In contrast to the investment into physical capital the formation of human capital is to a large extent not financed by its owner. Rather, parents and the state cover most of the expenditures on education. Parental engagement has been explained by credit market imperfections or by parental altruism (see, e.g. Becker and Tomes, 1986).

Parental altruism is traditionally assumed to be exogenously given, neglecting its source and evolutionary development. Among biologists and social-psychologists, on the other hand, there exists by now a large consensus that preferences, norms and cultural attitudes are endogenous with respect to our socio-economic system (see, e.g. Ehrlich, 2000). It is argued that two main channels exist through which preferences, values and norms of behavior are transmitted across generations. They are the outcome of a genetic and of a cultural transmission process. In fact, as argued by Heckman (2006), cognitive and non-cognitive skills, i.e. abilities and preferences, are influenced by an inextricable interaction between genetics and individual experience.

The transmission of the genotype obviously only runs from parents to children. Cultural transmission, on the other hand, is broader as it involves vertical, horizontal as well as oblique social relations. Cultural transmission is defined as the transmission of preferences, values and norms of behavior through social interaction. It implies a process of socialization and imitation of role models through which behavioral traits are transferred across and within generations (Bisin and Verdier, 2006). Although the concept of cultural transmission has a longstanding tradition in anthropology, sociology, and

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social psychology, research focusing on the endogenous formation and transmission of preferences has only more recently emerged in the economic literature (see, e.g. Bergstrom, 1996; Bisin and Topa, 2003; Hauk and Saez-Marti, 2002).

One area where the transmission of preferences through socialization and the imitation of role models has been found particularly important is the attitude to education. According to the socio-psychological literature parents have a pervasive influence in shaping young people's attitudes to education (see, e.g. Brooks, 2003; Davis-Kean, 2005; Cohen, 1987). [Cohen \(1987\)](#), for example, presents strong empirical evidence that parents influence their children's attitude and educational aspirations by being educational role models through their own educational achievements and by establishing standards that define what educational attainments are desirable.² As a consequence, children from parents with higher levels of education also tend to attach more importance to education themselves.

This hereditary transmission of the attitude to education has two aspects. On the one hand, parents' education is decisive for the importance they attribute to the education of their own children, and their willingness to finance their children's education. On the other hand, parents' education shapes the children's motivation and skills crucial for the children's education. With regard to the first aspect, the parents' attitude to the education of their children depends on the parents' own human capital, which in turn was shaped by the grandparents' attitude to the education of the parents, etc. In line with this, it has been found that parents' decision to finance their children's education strongly depends on their own education financing experience (e.g. [Steelman and Powell, 1991](#)). [Steelman and Powell \(1991\)](#), for example, show that parents whose college education was financed for by their parents are also much more likely to assist their children in financing their education. This aspect of the intergenerational transmission of education attitudes can also be viewed as an example of indirect reciprocity, which has been found to be particularly important within family relations (see, e.g. [Kohli and Künemund, 2003](#)). In contrast to direct reciprocity (see, e.g. [Dufwenberg and Kirchsteiger, 2004](#)), we speak of indirect reciprocity when a person does not directly reciprocate to the behavior of another person, but rather reciprocates indirectly to a third party (see, e.g. [Nowak and Sigmund, 1998](#); [Engelmann and Fischbacher, 2002](#)). In the context of education financing this means that the cultural transmission of preferences, values and norms across generations creates a pattern of behavior in which parents support their own children in a way similar to the way their parents treated them ([Steelman and Powell, 1991](#)). People do not directly reciprocate for the education they have received from their own parents, but rather repay it by financing the education of their children. Hence, the more education parents have received themselves, the more they are willing to finance the education of their children. In this way investments into human capital do not only affect the immediate recipient, i.e. the next generation, but also future generations.

The other aspect of the intergenerational transmission of education attitudes, namely the influence of parents' human capital on children's ability to acquire their own human capital, has already been extensively studied (see, e.g. [Eckstein and Wolpin, 1999](#); [Keane and Wolpin, 2001](#)). [Eckstein and Wolpin \(1999\)](#), for example, show that children of less educated parents have lower school ability and motivation, they place a higher value on leisure, and they have a lower value of consumption for school attendance (see also [Becker and Tomes, 1986](#); [Eckstein and Zilcha, 1994](#); [Galor and Tsiddon, 1997](#)). This empirical finding is the basis for the literature on the so-called 'home environment externality', where children's ability to acquire human capital depends on parental levels of education (see, e.g. [Becker and Tomes, 1986](#); [Eckstein and Zilcha, 1994](#); [Galor and Tsiddon, 1997](#)). It is assumed that the marginal product of investments into the human capital of children is increasing in the human capital level of parents. Hence, the higher the level of education of the parents, the more effective investments in human capital become. In line with this, [Keane and Wolpin \(2001\)](#) find empirical evidence that more educated parents make larger college attendance contingent financial transfers to their children. Furthermore, [Eckstein and Zilcha \(1994\)](#) analytically show that in the presence of such a home environment externality private investments into human capital are suboptimal, when parents only care about the educational level of their offspring.

Notwithstanding the empirical relevance of the home environment externality, this paper concentrates on the cultural transmission of the importance that parents attribute to the education of their children. Tractability considerations prevent us from explicitly incorporating the home environment externality into our model.³ But since the home environment externality strengthens the link between the parents' and the children's human capital, we suspect that the home environment externality would reinforce our results.

We investigate the impact of the cultural transmission of the importance parents attribute to the education of their children on human capital levels, incomes, well-being and education policies. In our setting the parents' preference for the human capital of their children depends on their own human capital, which in turn was financed by the grandparents. We incorporate this transmission of attitudes towards education into an OLG-model. Since there exists ample empirical evidence for an unequal distribution of human capital within societies and a strong correlation between the human capital levels of parents and children (see, e.g. [Bowles and Gintis, 2002](#); [Keane and Wolpin, 2001](#)), we allow for agents that are heterogeneous in their initial endowment as well as in their talent to acquire human capital.

We first analyze analytically the special case of an economy with a homogeneous population. In this case multiple steady states exist. There is always an 'illiterateness' steady state, where the whole society is characterized by low incomes and no investments into formal education. Depending on the parameters of the model, this 'illiterateness trap' can be unstable, locally stable, or globally stable.

² A similar intergenerational attitude transmission mechanism has been analyzed in the context of arts education (see [Champarnaud et al., 2005](#)).

³ In Section 2 we sketch how the home environment externality could be incorporated into our model.

To analyze the general case with a heterogeneous population, we have to use simulations. They show that an unequal initial distribution of human capital leads to steady states where an otherwise uniform population is permanently split into a group with large human capital endowment, large income, and high welfare, and a group with low human capital, low income, and low welfare. Depending on the initial conditions, the group with the low initial human capital might acquire no human capital in the long run. We also find that heterogeneities in the talent to acquire human capital lead to persistent differences in human capital, income, and welfare levels. The more heterogeneous the population is in terms of talents, the more likely it is that parts of society get stuck in an illiterateness trap. Our results indicate that the intergenerational transmission of education attitudes might lead to a permanent split of the society, where the economic and social status of an individual is mainly determined by the status of his parents. These results might explain the empirical evidence on the high intergenerational persistence of differences in life-time earnings as, e.g. shown by [Hendricks \(2007\)](#).

Finally, we analyze whether simple education policies can overcome the illiterateness trap and the permanent split of society. We find that compulsory schooling is a necessary condition to overcome illiterateness. We show that a simple tax-subsidy scheme can lead to a pareto-improvement over a situation with purely private financing of education. This analysis also sheds light on the political feasibility of such an education policy.

The paper is organized as follows. In the next section we describe the model. Section 3 presents the main results, namely the illiterateness trap and the permanent split of society. Section 4 analyzes education policy and welfare. Finally, we draw conclusions. All the proofs are delegated to the Appendix.

2. The model

The analysis is based on an OLG-model of a competitive economy with three overlapping generations. Take an individual born at the beginning of period $t - 1$. In this period he belongs to the youngest generation 1 which gets educated. The amount of his education is decided upon by his parent. In the next period t , the individual belongs to the working (parent) generation 2. In this period he works and has one child.⁴ He divides his income between consumption in period t , savings for consumption in $t + 1$ and spending for the education of his child. In period $t + 1$ the individual belongs to the retired generation 3. His previous savings are now used for production, and he consumes the rent on his savings. At the end of period $t + 1$, the individual dies.

Generation	Period		
	$t - 1$	t	$t + 1$
(1)	Education		
(2)		Work	
(3)			Retirement

For simplicity, we assume that in each period each generation consists of the same number of individuals, denoted by L . The total population is split into two groups, A and B . The share s of A -types is the same in each generation and in each period. In period t a representative worker of type I , $I = A, B$, exhibits a human capital of h_t^I . In period t a representative retiree of type I , $I = A, B$, owns a physical capital of k_t^I .

Overall output in period t , Y_t , depends on overall physical capital, $L(sk_t^A + (1 - s)k_t^B)$, and on the overall human capital, $L(sh_t^A + (1 - s)h_t^B)$, available in t . The economy is endowed with a Cobb–Douglas production technology. The normalized production function is given by

$$y_t = \left(sk_t^A + (1 - s)k_t^B \right)^\alpha \left(sh_t^A + (1 - s)h_t^B \right)^{1-\alpha}, \quad (1)$$

where $\alpha \in (0, 1)$ and $y_t = Y_t/L$ denotes the output per worker.

Markets are assumed to be perfectly competitive. This implies an interest rate r_t of

$$r_t = \frac{\partial y_t}{\partial (sk_t^A + (1 - s)k_t^B)} = \alpha \left(\frac{sh_t^A + (1 - s)h_t^B}{sk_t^A + (1 - s)k_t^B} \right)^{1-\alpha}. \quad (2)$$

The capital stock depreciates fully in one period, so that the savings in period $t - 1$ equal the capital stock in period t .

The effective wage rate per unit of human capital is given by

$$\frac{\partial y_t}{\partial (sh_t^A + (1 - s)h_t^B)} = (1 - \alpha) \left(\frac{sk_t^A + (1 - s)k_t^B}{sh_t^A + (1 - s)h_t^B} \right)^\alpha. \quad (3)$$

⁴ For simplicity we assume that each adult has only one child, and each child has only one parent.

Agents receive the same effective wage rate and interest rate. Furthermore, we assume that every worker supplies inelastically one unit of labor. However, given the heterogeneity in the population, workers may differ in the wage that they earn as they may differ in their amount of human capital. Wages are given by

$$w_t^I = h_t^I (1 - \alpha) \left(\frac{sk_t^A + (1-s)k_t^B}{sh_t^A + (1-s)h_t^B} \right)^\alpha, \quad I = A, B. \quad (4)$$

Young individuals belonging to generation 1 are formally educated, i.e. go to school, in order to acquire human capital. But even without any formal education agents are productive to some extent, i.e. they possess some minimum human capital even without schooling. Furthermore, the two groups of agents differ in their ability to acquire human capital. Taking this into account the production function for human capital is given by

$$h_{t+1}^I = \eta^I ((e_t^I)^\beta + 1), \quad I = A, B, \quad (5)$$

$\eta^I > 0$ measures the talent of a child of type I to acquire human capital. $e_t^I \geq 0$ denotes the parent's expenditures into the formal education of a child of type I born in t . Of course, the resulting human capital becomes productive in period $t + 1$. Note that Eq. (5) implies that each type has a minimum human capital level of η^I . So even without formal education the more talented agents are more productive than less talented ones. Without any loss of generality, we denote the (weakly) more talented type by A , i.e. $\eta^A \geq \eta^B$.⁵

We assume decreasing returns to expenditures in schooling, i.e. $\beta \in (0, 1)$. Otherwise, the model might not exhibit steady states. In a previous version of the paper (see Kirchsteiger and Sebald, 2006) we showed analytically for the special case of a homogeneous population that the existence of a steady state can only be guaranteed for $\beta < 1$. Similarly, simulations suggest that with $\beta \geq 1$ no steady state exists for the general heterogeneous population case.

We have assumed that s , i.e. the fraction of A -types, remains constant over time. In order to guarantee this, we assume that each child inherits the talent of his parent. Clearly, this assumption is simplifying, since it excludes mobility between talent groups. But inheritance of talent can be justified to some extent by the empirical evidence that a substantial part of the intelligence of human beings is genetically determined (see, e.g. Bowles and Gintis, 2002; DeFries et al., 2000). So while talent might be random, evidence shows that parent's and child's talents are strongly positively correlated. Modeling such a random process would significantly complicate the analysis. But we suspect that random inheritance of the talent would lead to the same qualitative results, provided that the intergenerational correlation of talents is strong enough.

Note that such a direct, genetic inheritance of talent differs from the home environment externality, where the talent to acquire human capital depends on the human capital of the parents. To model the home environment externality, one would have to introduce an increasing talent function $\eta_{t+1}(h_t^I)$. As already explained in the Introduction, we regard the home environment externality as another important aspect for transmitting human capital levels from generation to generation. However, for tractability reasons we restrain from modeling it explicitly. We suspect that introducing it into the analysis would only strengthen our results.

Only individuals belonging to the working generation have to make a decision. I -type workers in period t maximize their utility function given by

$$U^I(c_{2,t}^I, c_{3,t+1}^I, h_{t+1}^I) = \ln c_{2,t}^I + \gamma \ln c_{3,t+1}^I + \varphi^I(h_t^I) \ln h_{t+1}^I, \quad (6)$$

where $c_{2,t}^I$ denotes the immediate consumption of an individual of type I working in period t . $c_{3,t+1}^I$ is the consumption in the next period $t + 1$ when the individual belongs to the retired generation 3. Since we assume full depreciation of the capital stock in one period, the savings in period t are the capital stock in period $t + 1$, and the old generation only consumes the interest on their savings. Therefore, $c_{3,t+1}^I = k_{t+1}^I r_{t+1}$. h_{t+1}^I is the human capital of the child, which becomes effective in period $t + 1$. γ and φ measure the individual's attitude towards future old-age consumption and towards the human capital of the child, respectively.

In order to formalize the transmission of preferences, values and norms of behavior in the area of education financing, we abstract from the micro-process describing the socialization and imitation of role models. We simply start from the empirical findings described in the introduction that the education a parent has received in his own childhood shapes his willingness to invest into the human capital of his own child. We capture this relation by a continuous and differentiable attitude function which describes how the education of parents relates to the importance they attach to the education of their children:

$$\varphi^I : [\eta^I, \infty) \rightarrow \mathfrak{R}_+^0,$$

with $\varphi^I(h_t^I)$ denoting the attitude of a parent of type I with a human capital of h_t^I .⁶ The attitude towards children's education is positively correlated with parents' human capital h_t^I , which in turn was financed for by the grandparent. Furthermore, if

⁵ Alternatively, one might assume that the minimum human capital level is the same for both types even if they differ in their talent acquire formal education. A human capital production function like $h_{t+1}^I = \eta^I (e_t^I)^\beta + 1$ would exhibit this feature. None of our results would change qualitatively with this alternative approach.

⁶ In this formulation we allow for type specific attitude functions. Alternatively, we could have assumed that both types are endowed with the same attitude function. All of our qualitative results would also hold within such a framework.

the parent has not received any formal education himself, he is not willing to finance any formal education of his child. Formally,

$$\frac{\partial \varphi^I(h_t^I)}{\partial h_t^I} > 0,$$

$$\varphi^I(\eta^I) = 0.$$

Agents of type I working in period t have to decide how much of their wage income w_t^I they want to spend on instantaneous consumption and on the education of their child. Furthermore, they save in order to finance consumption when they are retired. Recall that $e_t^I = ((h_{t+1}^I/\eta^I) - 1)^{1/\beta}$, and due to full depreciation of the capital stock, $c_{3,t+1}^I = k_{t+1}^I r_{t+1}$.

The maximization problem of an agent of type I working in t is given by

$$\begin{aligned} \max_{c_{2,t}^I, k_{t+1}^I, h_{t+1}^I} \quad & U(c_{2,t}^I, k_{t+1}^I, h_{t+1}^I) = \ln c_{2,t}^I + \gamma \ln k_{t+1}^I r_{t+1} + \varphi^I(h_t^I) \ln h_{t+1}^I \\ \text{s.t.} \quad & w_t^I = c_{2,t}^I + k_{t+1}^I + \left(\frac{h_{t+1}^I}{\eta^I} - 1 \right)^{1/\beta}, \\ & h_{t+1}^I \geq \eta^I, \\ & c_{2,t}^I, k_{t+1}^I \geq 0. \end{aligned} \tag{7}$$

For a period t with given human and physical capital levels the competitive equilibrium is characterized by the wage equation and the solution to the maximization problem.⁷ Formally,

Definition 1. For given $k_t^I, h_t^I, I = A, B$, the competitive equilibrium of the economy in period t is given by $w_t^I, c_{2,t}^I, k_{t+1}^I, h_{t+1}^I, I = A, B$ such that (i) Eq. (4) holds and (ii) the maximization problem (7) is solved.

Denote by $(\tilde{k}_t^I, \tilde{h}_t^I)_{t=2}^\infty, I = A, B$, the sequence of the equilibrium values of human and physical capital, and by $k_1^I, h_1^I, I = A, B$, the initial endowments with physical and human capital. The first order conditions (FOCs) of the solution of the utility maximization problem are given by

$$\begin{aligned} \frac{\partial U}{\partial h_{t+1}^I} &= \frac{\varphi^I(\tilde{h}_t^I)}{\tilde{h}_{t+1}^I} - \frac{(\eta^I)^{-1/\beta}}{\beta} (\tilde{h}_{t+1}^I - \eta^I)^{(1/\beta)-1} \\ \tilde{w}_t^I - \tilde{k}_{t+1}^I - \left(\frac{\tilde{h}_{t+1}^I}{\eta^I} - 1 \right)^{1/\beta} &= 0, \quad I = A, B \end{aligned} \tag{8}$$

and

$$\frac{\partial U}{\partial k_{t+1}^I} = \frac{\gamma}{\tilde{k}_{t+1}^I} - \frac{1}{\tilde{w}_t^I - \tilde{k}_{t+1}^I - \left(\frac{\tilde{h}_{t+1}^I}{\eta^I} - 1 \right)^{1/\beta}} = 0, \quad I = A, B. \tag{9}$$

As Lemma 1 in the Appendix shows, these FOCs and the wage Eq. (4) fully characterize the sequence of competitive equilibria whenever $k_1^A + k_1^B > 0$. If $k_1^A + k_1^B = 0$, no production, no consumption, and no formal education is possible in any future period. Since this case is not interesting, we restrict the analysis from now on to initial conditions with $k_1^A + k_1^B > 0$.

3. The illiterateness trap and the split of society

In this section we present the main results of the model. In particular, we show how the transmission of the attitude towards education can lead to an illiterateness trap and to a permanent split of society. The illiterateness trap can most easily be understood for the special case of a homogeneous population, which we will analyze first. Then we investigate the general case of a heterogeneous population, where the illiterateness trap might split the population permanently.

3.1. The economy with a homogeneous population

In order to model a homogeneous population, we set $s = 0$ and drop the superscripts referring to the type. Without loss of generality human capital is measured such that the minimum human capital level is one, i.e. $\eta = 1$. It is well known that for a fixed and strictly positive level of parental attitude towards children's education, i.e. for an exogenously given $\varphi > 0$,

⁷ Recall that in period t the physical and human capital levels of the next period, k_{t+1}^I and h_{t+1}^I are determined.

there exists a unique interior steady state where the whole population has the same human capital level $h^* > 1$. Furthermore, simulations suggest that this steady state is globally stable.⁸ So if the parents' attitude towards children's education does not depend on their own human capital, children will get formal education, and different initial endowments of human capital will be equalized in the long run.

These results contrast sharply with those of an economy with an endogenous education attitude, i.e. of an economy where the education attitude depends on the parents' human capital. Note that $\varphi(1) = 0$ because $\eta = 1$. This implies that conditions (8) and (9) are always fulfilled by

$$\tilde{h}_t = \tilde{h}_{t+1} = h^* = 1,$$

$$\tilde{k}_t = \tilde{k}_{t+1} = k^* = \left(\frac{\gamma(1-\alpha)}{1+\gamma} \right)^{1/(1-\alpha)}.$$

In this steady state, no formal education takes place, and human capital is at its lowest possible level. This steady state, which we denote as illiterateness steady state, characterizes a situation where the whole population is trapped in a vicious chain in which formal education is neglected: Since parents have no formal education, they are not willing to finance the formal education of their children, and hence the children are not interested in the education of the grandchildren, and so on.

Whether this illiterateness trap poses a severe problem depends crucially on the stability properties of this steady state and on the existence of other steady states. The stability properties of the illiterateness steady state as well as the existence and the properties of other steady states depend on the form of the attitude function, $\varphi(h_t)$. To illustrate the different possible outcomes, we use a simple attitude function, namely

$$\varphi(h_t) = \frac{1}{\delta}(h_t - 1). \quad (10)$$

Using this attitude function we get the following:

Proposition 2. (i) If $\beta < \frac{1}{2}$, there exists exactly one interior steady state with formal education in addition to the illiterateness steady state.

(ii) If $\beta > \frac{1}{2}$, the following holds: Except for non-generic values of the parameters of the model, there exist either two or no interior steady states with formal education in addition to the illiterateness steady state.

Proof. See the Appendix.

To analyze the stability properties of the illiterateness steady state we run simulations using the wage equation (4) and the FOCs (8) and (9).⁹ They show that for $\beta < \frac{1}{2}$ the illiterateness steady state is unstable. For $\beta > \frac{1}{2}$ the illiterateness steady state is globally stable when no interior steady state exists. If two interior steady states exist, one of them as well as the illiterateness steady state are locally stable. The intuitive reason for this result is straightforward. For the analysis of the stability of the illiterateness trap education expenditures in the vicinity of zero are relevant. The human capital production function (5) reveals that for education expenditures below 1 the achieved human capital level decreases in β . Hence, a lower β makes it easier to escape the illiterateness trap.

The possible multiplicity of stable steady states refers to whole economies: Depending on the initial conditions, otherwise identical societies might end up at different steady states (and connected welfare levels). One wonders whether it is possible that within a given economy the population might be split permanently: If the initial endowment with human capital is different for otherwise identical members of the first generation, will their descendants end up at different education and utility levels? If the members of society differ in their talent to acquire human capital, is it possible that part of the population gets stuck in the illiterateness trap? In order to answer these questions, we have to investigate the general case of an economy with a heterogeneous population.

3.2. The economy with a heterogeneous population

The properties of the general model with a heterogeneous population cannot be analyzed analytically. Consequently, we will run simulations using an attitude function similar to Eq. (10), namely

$$\varphi^I(h_t^I) = \frac{1}{\delta}(h_t^I - \eta^I) \quad \text{for } I = A, B. \quad (11)$$

We first look at the case of a population characterized by heterogeneous initial endowments in human capital, but similar talents $\eta^A = \eta^B = \eta$. Clearly, whenever $\tilde{k}_t^A = \tilde{k}_t^B$ and $\tilde{h}_t^A = \tilde{h}_t^B$ the FOCs of the two types are the same. Hence, any steady state of the model with a homogeneous population constitutes also a steady state of the heterogeneous population model when talents do not differ. But even if talents are the same, there may also exist steady states where the population

⁸ A formal analysis of this claim can be found in a previous version of this paper (Kirchsteiger and Sebald, 2006).

⁹ Details of these simulations are available from the authors upon request.

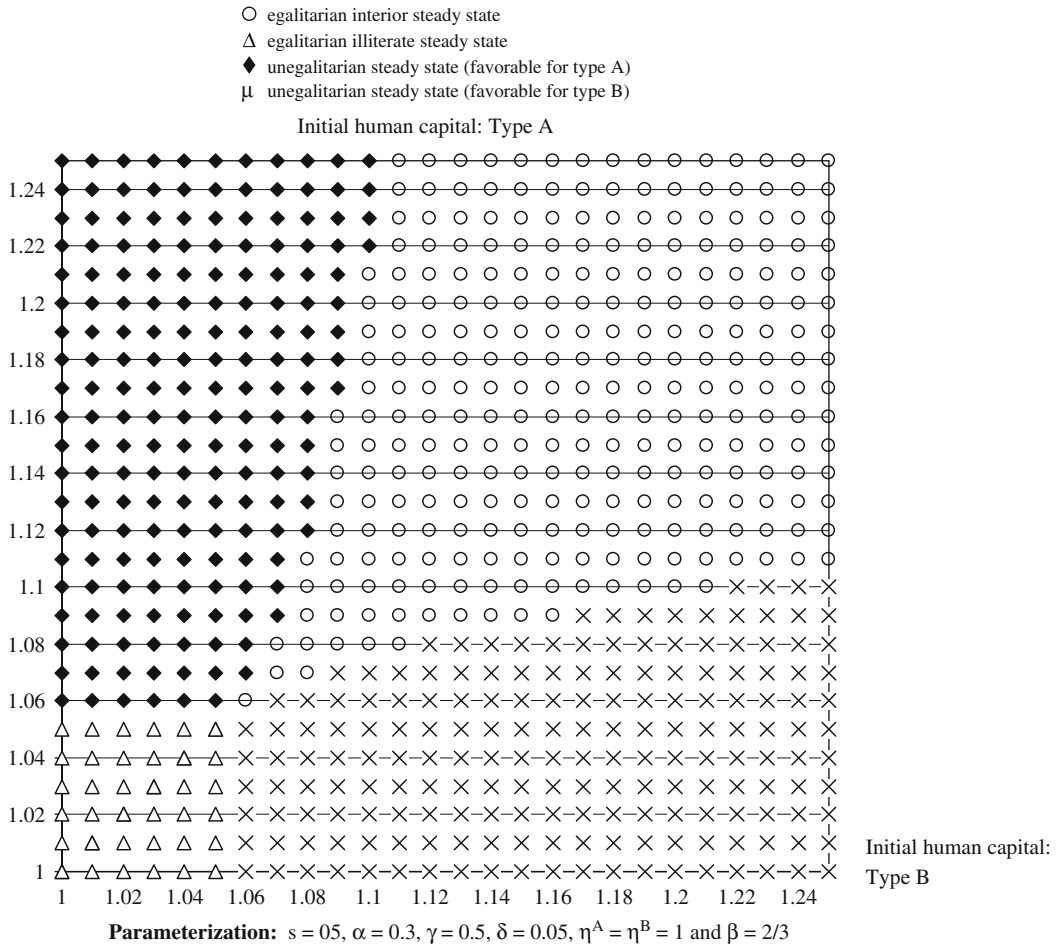


Fig. 1. Heterogeneous initial levels of human capital: $s = 0.5$, $\alpha = 0.3$, $\beta = \frac{2}{3}$, $\gamma = 0.5$, $\delta = 0.05$, and $\eta^A = \eta^B = 1$.

remains split into two groups even in the long run. Take, for example, the model with the following parameter values: $s = 0.5$, $\alpha = 0.3$, $\beta = \frac{2}{3}$, $\gamma = 0.5$, $\delta = 0.05$, and $\eta = 1$. This model specification exhibits two locally stable egalitarian steady states where both types have the same human and physical capital level, income, etc. One of these egalitarian steady states is the illiterateness trap for the whole population with $h^* = 1$, $k^* = 0.125$, and realized utility of $U^* = -2.425$. The other egalitarian steady state is interior with $h^* = 1.237$, $k^* = 0.095$, and realized utility of $U^* = -1.823$. But on top of these two egalitarian steady states there are two inegalitarian steady states. One of them is favorable for type A agents with $h^{A*} = 1.317$, $k^{A*} = 0.088$, and $h^{B*} = 1$, $k^{B*} = 0.112$, leading to realized utility levels of $U^{A*} = -1.208$ and $U^{B*} = -2.593$. In this inegalitarian steady state a part of the population, namely the B-types, get stuck in the illiterateness trap, whereas the A-types have higher human capital, higher wages, and consequently higher utility levels. The other inegalitarian steady state is favorable for type B agents. It is symmetric to the inegalitarian steady state favorable to type A agents.

Which of the four steady states is reached depends on the initial distribution of human capital. To get a better understanding, consider the simulation results summarized in Fig. 1, which are calculated for initial physical capital levels of 1 for both groups.¹⁰

In Fig. 1 we show for which initial human capital levels h_1^A , h_1^B , the process converges to the interior egalitarian, to the egalitarian illiterate, or to one of the inegalitarian steady states. For very low initial levels of human capital for both types of agents, both types of agents fall into the illiterateness trap ($h^{A*} = h^{B*} = 1$ and $k^{A*} = k^{B*} = 0.125$). For high and similar initial levels of initial human capital, both types of agents avoid the illiterateness trap in the long run, and the system converges to the interior egalitarian steady state with $h^* = 1.237$ and $k^* = 0.095$. If the initial endowment with human capital is high for A-types and low for B-types, the two groups differ even in the long run. The steady state reached is inegalitarian, with only the B-types being trapped in illiterateness. Symmetrically, only the A-types are illiterate in the long run, when A-types are

¹⁰ Obviously the initial distribution of physical capital also determines which steady state is reached. Since we are mainly interested in human capital formation, we concentrate on differences in initial human capital endowments. Note, however, that a higher initial physical capital translates into a higher human capital level in the next period. Hence, simulations with different initial physical capital levels would give qualitatively similar results.

initially endowed with little and the *B*-types with a lot of human capital. These results are very intuitive—the higher the initial human capital, the less likely the economy gets stuck in the illiterateness trap, and the higher the difference in initial human capital, the more likely it is that only the disadvantaged type gets stuck in illiterateness.

The importance of these results depends of course crucially on whether they hold for generic economies. So one wonders how sensitive these results are with respect to (small) variations of parameters. To answer this question, we conducted simulations where we, respectively, varied the value of one of the parameters α , β , γ , δ , η , and s . These simulations are based on the baseline parametrization: $s = 0.5$, $\alpha = 0.3$, $\beta = \frac{2}{3}$, $\gamma = 0.5$, $\delta = 0.05$, $\eta = 1$, and $k_1^A = k_1^B = 1$. The results of these parameter variations also provide some (local) comparative statics. Space limitation prevent us from presenting the results of the underlying simulations in detail. We can only report some summary results. The detailed simulation results are available from the authors upon request.

We find that the main result, namely the existence of all four types of steady states, is locally robust to changes of α , β , γ , δ , η , and s . Furthermore, all the parameters have the implications to be expected. An increase in α , or in δ , and a decrease in η enlarges set of initial endowment levels for which an egalitarian as well as an inegalitarian illiterateness trap is reached. For higher α , people save more and invest less in the education of their children. A higher δ is equivalent to a decrease in the parents' willingness to finance the education of their children, and a lower η means that agents are less talented in acquiring human capital. In all three cases either one type (for unequal initial endowments) or both types (for similar low initial endowments) are more prone to the illiterateness trap. The same result holds for an increase in β . To understand this, note that close to the illiterateness trap the education expenditures are close to 0. For education expenditure levels below 1 an increase in β implies that higher expenditures are necessary to reach the same level of human capital. Hence, it becomes more likely that a part or the whole economy cannot overcome illiterateness if β is higher.

The likelihood of an egalitarian interior steady decreases in γ , the importance of old age consumption. We could not detect any impact of γ on the likelihood of the illiterate egalitarian steady state. An increase of γ leads to a decrease in spendings on the children's education, making it more likely that the disadvantaged group becomes illiterate if the initial endowment is already very uneven.

Finally, we investigate the impact of the share s of *A*-type. If the *A*-types are richer in initial human capital than the *B*-types, an egalitarian interior steady state is more likely the lower s . So the smaller the share of people initially rich in human capital, the more likely it is that the underdogs (in this case the *B*-types) can avoid illiterateness. The more people rich in human capital are in the economy, the lower is the wage rate, and consequently the lower is the income of the underdogs. Hence, it becomes less likely that the underdogs can avoid illiterateness.

This discussion shows that for generic parameters it is possible that the long run differences in human capital, incomes and welfare prevail, even if all agents are equally talented and factor markets are perfectly competitive. Illiterateness gets inherited from generation to generation, preventing convergence of the two population groups.

Next we consider the situation in which the two groups in the population do not differ in terms of initial conditions, but in terms of their talent to acquire human capital, i.e. $\eta^A \neq \eta^B$. Recall that this also implies that the minimum human capital level of the two types differs. In this case the steady state values of human capital, wages, etc. might differ even if both groups start at the same initial value of human and physical capital, provided that the initial endowment with human and/or physical capital is large enough. Take as an example the economy with $s = 0.5$, $\alpha = 0.3$, $\beta = \frac{2}{3}$, $\gamma = 0.5$, $\delta = 0.05$, $k_1^A = k_1^B = 1$, and $\eta^A = 1.1$, $\eta^B = 1$. If the initial endowment with human capital is relatively small, e.g. $h_1^A = 1.11$ and $h_1^B = 1.01$, the egalitarian illiterateness steady state with $h^{A*} = 1.1$ and $h^{B*} = 1$ results. If the initial endowment with human capital is relatively large, e.g. $h_1^A = h_1^B = 1.25$, an inegalitarian steady state with $h^{A*} = 1.530$ and $h^{B*} = 1$ results—despite the same initial endowment with human capital, the less talented *B*-types converge to illiterateness, whereas the more talented *A*-types prosper in the long run.

Table 1
Heterogeneous ability.

Talent.		Steady state human capital	
A	B	A	B
1.01	1	1.34004	1
1.009	1	1.3378	1
1.008	1	1.33555	1
1.007	1	1.3333	1
1.006	1	1.33104	1
1.005	1	1.32877	1
1.004	1	1.3265	1
1.003	1	1.32423	1
1.002	1	1.32194	1
1.001	1	1.2485	1.22688
1.0005	1	1.2428	1.23241
1	1	1.23721	1.23721

The larger the difference in talent, the more likely it is that the *B*-types are illiterate. Take Table 1 which has been calculated for the economy with $s = 0.5$, $\alpha = 0.3$, $\beta = \frac{2}{3}$, $\gamma = 0.5$, $\delta = 0.05$, $k_1^A = k_1^B = 1$, $h_1^A = h_1^B = 1.25$, and $\eta^B = 1$.

Only for very small differences in talent ($\eta^A < 1.002$), both types of agents acquire formal education in the long run, with the less talented *B*-types acquiring less human capital. In this case the steady state is inegalitarian, but no group is illiterate. If the difference in talent is larger ($\eta^A \geq 1.002$) the *B*-types are illiterate in the long run. As to be expected, Table 1 also shows that the steady state difference increases in the differences in talent.

Finally, agents may differ in initial human capital as well as in talent. If the agents of one type, say *A*, are initially richer as well as more talented, then it is more likely that the *B*-types end up in illiteracy. To see this take again the model with the parameter values $s = 0.5$, $\alpha = 0.3$, $\beta = \frac{2}{3}$, $\gamma = 0.5$, $\delta = 0.05$, and $k_1^A = k_1^B = 1$. For $h_1^A = 1.25$, $h_1^B = 1.11$, and $\eta^A = \eta^B = 1$, i.e. if both types differ only in the initial endowment, an interior egalitarian steady state results with *B*-types avoiding the illiterateness trap (see Fig. 1). For $h_1^A = h_1^B = 1.25$, $\eta^A = 1.001$, and $\eta^B = 1$, i.e. if both types differ only in their talents, an inegalitarian, but nonetheless interior steady state occurs (see Table 1), where *B*-types are not illiterate in the long run. But if the differences in initial endowment and talent get combined, i.e. for $h_1^A = 1.25$, $h_1^B = 1.11$, $\eta^A = 1.001$, and $\eta^B = 1$, the steady state is characterized by $h^{A*} = 1.34$ and $h^{B*} = 1$. Hence, the combined effect of differences in initial endowment and talent makes the illiterateness trap more likely. Of course, the opposite conclusion holds if the initially richer types are less talented.

4. Education policy and welfare

Till now we have assumed that the state does not intervene into the private decision of the parents about the education of their children. In reality, however, education policies are an important part of the political process. So in this section we want to analyze whether and how education policies can overcome the illiterateness trap and achieve efficiency improvements. Within our model parents do not care about the welfare of their children, but only about their human capital. The effect of the parents' decision on the welfare of the children is not fully internalized, and consequently even interior steady states, where the whole population gets formal education, are not pareto-efficient. In principle, this externality based inefficiency could be overcome by a subsidy for education financed by a non-distortionary tax. However, efficient tax-subsidy schemes would have to change from period to period, and they would have to be different for different groups of society. This means, efficient tax-subsidy schemes would require a lot of information, and they would have to be quite flexible.¹¹ Furthermore, different efficient tax-subsidy schemes would lead to differences in the inter- as well as intragenerational distributions of welfare, and the choice of a particular efficient scheme would give rise to inter- as well as intragenerational conflicts.

For all these reasons, it seems doubtful whether the actual political process could provide such a complicated and flexible optimal scheme. Therefore, we restrict the analysis to simple, often observed, and relatively easy to implement education policies. In most countries compulsory schooling and a public financing of some of the education costs are basic features of the education system. Therefore, in this section we investigate whether and when a combination of these two policies can overcome the illiterateness trap, and whether they can lead to pareto-improvements compared to situations where the education level is exclusively decided and financed by parents.

In our context, compulsory schooling can be seen as equivalent to compulsory minimum expenditures for the children's education. We assume that the state does not or cannot distinguish between the two types, and hence parents of both types are subject to the same minimum education expenditures. For reasons of expositional ease, we will not use the minimum education expenditures as an explicit policy variable. We use instead the human capital levels resulting from compulsory schooling, i.e. from the minimum education expenditures. So denote by h_c^l the human capital level of a child of type *l* with compulsory schooling. As the human capital production function (5) reveals, the same level of minimum education expenditures for both types implies that $h_c^A = (\eta^A/\eta^B)h_c^B$. So whenever the talents are different, compulsory schooling leads to different human capital levels.¹² Furthermore, for all the results reported below, h_c^l is set low enough such that all parents can afford to finance the associated expenditures.

Most of the education subsidies are not given in cash, but as non-cash benefits. At least partly the state finances schools, universities and other education institutions. Obviously, the more educated the children, the more the parents benefit from these non-cash benefits—the subsidies increase in the education level. Note further that these subsidies typically cover only parts of the education costs. Even if schools and universities are fully financed by the state, parents still have to take care of the children's costs of living, the costs of supplementary education, the costs of teaching material and other things indirectly connected to the human capital formation of children. In such a system of mixed financing a better education of the children requires higher expenditures of parents as well as of the state. Finally, the education level of the children is

¹¹ In a previous version of the paper (see Kirchsteiger and Sebald, 2006) we characterize the optimal tax-subsidy schemes for the case of a homogeneous population.

¹² Compulsory schooling could also be modeled as guaranteeing the same minimum human capital level for both types of children, even if they exhibit differences in talent. Our results would not change qualitatively by using this alternative formulation. However, this Approach seems less realistic. Compulsory schooling does not take the form that a child has to attend school until he has acquired certain skills. Rather, compulsory schooling typically requires that children attend school for a certain number of years. And one can plausibly expect that the more talented a child, the greater the human capital he acquires during this compulsory school years.

largely influenced by the parent's willingness to cover the children's costs of living, even when the state finances schools and universities. To model such a subsidy system, we assume that the private expenditures on the education of both types get subsidized at the same rate, and that the subsidy rate is constant over the periods. Since the subsidy never covers all the costs, the subsidy rate σ is assumed to be strictly below 1. To finance the subsidy wage income is taxed. The tax rate is the same for both groups, but varies over the periods in order to guarantee that the budget is balanced in every period.¹³ Denoting the tax rate by τ_t the balanced budget condition for period t reads

$$\sigma \left[s \left(\frac{h_{t+1}^A}{\eta^A} - 1 \right)^{1/\beta} + (1-s) \left(\frac{h_{t+1}^B}{\eta^B} - 1 \right)^{1/\beta} \right] = \tau_t [s w_t^A + (1-s) w_t^B]. \quad (12)$$

We assume that an individual agent takes the tax rate and the subsidy scheme as given when he maximizes his utility. This implies that he does not take into account the balanced budget condition of the state. This assumption seems plausible for a large economy with many agents. Using the attitude function (11), the decision problem of an agent of type $I = A, B$ who works in period t can be written as

$$\begin{aligned} \max_{c_{2,t}^I, k_{t+1}^I, h_{t+1}^I} \quad & U^I(c_{2,t}^I, c_{3,t+1}^I, h_{t+1}^I) = \ln c_{2,t}^I + \gamma \ln k_{t+1}^I r_{t+1} + \frac{1}{\delta} (h_t^I - \eta^I) \ln h_{t+1}^I \\ \text{s.t.} \quad & (1 - \tau_t) w_t^I = c_{2,t}^I + k_{t+1}^I + (1 - \sigma) \left(\frac{h_{t+1}^I}{\eta^I} - 1 \right)^{1/\beta} \quad \text{and} \\ & h_{t+1}^I \geq h_t^I, \\ & c_{2,t}^I \geq 0, \\ & c_{3,t+1}^I \geq 0. \end{aligned} \quad (13)$$

For period t with given levels of human and physical capital the competitive equilibrium of the economy with education policy is characterized by the solution to the maximization problem, the wage equation, and the balanced budget condition. Formally,

Definition 3. For given $k_t^I, h_t^I, I = A, B$, the competitive equilibrium of the economy with education policy in period t is given by $\tau_t, w_t^I, c_{2,t}^I, k_{t+1}^I, h_{t+1}^I, I = A, B$ such that (i) Eq. (12) holds, (ii) Eq. (4) is fulfilled, and (iii) the maximization problem (13) is solved.

If the maximization problem (13) has an interior solution with $h_{t+1}^I > h_t^I$ the following first order conditions are fulfilled:

$$\begin{aligned} \frac{\partial U^I}{\partial h_{t+1}^I} &= \frac{1}{\delta} \frac{(h_t^I - \eta^I)}{h_{t+1}^I} - \frac{(1 - \sigma) \frac{(\eta^I)^{-1/\beta}}{\beta} (\tilde{h}_{t+1}^I - 1)^{(1/\beta)-1}}{(1 - \tau_t) \tilde{w}_t^I - \tilde{k}_{t+1}^I - (1 - \sigma) \left(\frac{\tilde{h}_{t+1}^I}{\eta^I} - 1 \right)^{1/\beta}} = 0, \quad I = A, B, \\ \frac{\partial U^I}{\partial k_{t+1}^I} &= \frac{\gamma}{\tilde{k}_{t+1}^I} - \frac{1}{(1 - \tau_t) \tilde{w}_t^I - \tilde{k}_{t+1}^I - (1 - \sigma) \left(\frac{\tilde{h}_{t+1}^I}{\eta^I} - 1 \right)^{1/\beta}} = 0, \quad I = A, B. \end{aligned}$$

We cannot characterize analytically the solution of the maximization problem. Rather, we use the FOCs, the $h_{t+1}^I \geq h_t^I$ constraints, the balanced budget condition and the wage equation to simulate the impact of the policy measures. We investigate an economy with an homogeneous population first, and then the heterogeneous population economy. Before we turn to the detailed analysis, note that the illiterateness trap can never be overcome by a subsidy alone. The reason is that without compulsory schooling the uneducated parents will not invest in the education of the children for any subsidy rate below 1. Hence, we assume for the rest of this section that there is at least some compulsory schooling, i.e. $h_c^B > \eta^B$, which implies that $h_c^A > \eta^A$.

Note further that the education policy measures do not only have an impact on the human capital. They also influence the amount of savings, and by that the amount of physical capital available to the economy. As can be seen from the maximization problem (13), more compulsory schooling (if it is binding) and a higher subsidy rate decrease the amount of savings for a given wage. So the question arises whether the overall effect of education measures increases welfare. This will be discussed in detail in the next two sections.

¹³ This formulation assumes that tax rates are more flexible than subsidy rates. This implicit assumption can be justified by the fact that most of the time subsidies are not paid directly in cash. Rather, they take the form of salaries for the teachers, construction and maintenance costs of schools, etc. Typically these expenditures are more difficult to adjust than tax rates. Our main results, however, would not change if we allow for flexible subsidy rates and constant tax rates.

4.1. Education policy for a homogeneous population economy

As in Section 3.1 the economy with a homogeneous population is modeled by $s = 0$ and $\eta^B = 1$. To evaluate the impact of education policy we simulate the long term convergence levels of human capital and utility when the simulation process starts at the illiterateness steady state.¹⁴

If the illiterateness trap is unstable, i.e. for $\beta < \frac{1}{2}$, compulsory schooling at any level is sufficient to overcome the illiterateness trap. Furthermore, for low subsidy rates the steady state utility is increasing in the subsidy rate level. In this case the education subsidy mitigates the problem that parents internalize children's welfare only insufficiently. But for higher subsidies rates this positive internalization effect is more than counterbalanced by the negative distortive effect of subsidies on the decision between own consumption and children's education.

If the illiterateness trap is globally stable, and if there are no education subsidies on top of compulsory schooling, the human capital level will always remain at h_c^B . In this case compulsory schooling increases the level of human capital, but the population will not get more educated than the level enforced by compulsory schooling. The simulations also show that with relatively high subsidies ($\sigma \geq \frac{1}{2}$) a self-supporting education process can be started, where compulsory schooling is not binding in the long run. But because of distortionary effects the long run welfare with such high subsidies is lower than the welfare resulting from low or zero subsidy rates, when the human capital is only at the level enforced by compulsory schooling.

Finally, there is the possibility of an economy exhibiting a locally stable illiterateness trap and a locally stable interior steady state. Again, without compulsory schooling the population stays illiterate forever if it gets trapped in illiterateness. Furthermore, for a low level of compulsory schooling, and without education subsidies, there will be no education on top of compulsory schooling. Again, subsidies might start a self-supporting education process. And unlike the case with a globally stable illiterateness trap, an interior steady state with steady state education levels above the compulsory level might be welfare improving.

So the education policy necessary to overcome the illiterateness of the whole population depends crucially on the stability characteristics of the illiterateness trap which, in turn, depends on the human capital production technology. If β is low, low levels of compulsory schooling are sufficient to overcome illiterateness, and the compulsory schooling constraint will not be binding in the long run.¹⁵ If β is high, the illiterateness trap is (locally or globally) stable, and higher levels of compulsory schooling are necessary to reach an interior steady state. Furthermore, in some cases an education subsidy can lead to a further welfare improvement, since it has the same effect as a (partial) internalization of children's welfare into parents utility.

4.2. Education policy with a heterogeneous population

If the population is heterogeneous there is the possibility that there is no convergence of the different types even in the long run, and parts of society might be caught in illiterateness. There are two reasons for such inegalitarian outcomes. On the one hand, the initial endowment with human capital might be very uneven. On the other hand, the talent to acquire human capital might be unequally distributed. The impact of the education policy depends on the type of heterogeneity.

We have seen in Section 3.2 that if both types are equally talented and if the initial endowments of the different types is not too unequal, an egalitarian steady state is realized. The heterogeneous population actually becomes homogeneous in the long run. Consequently, the education policy needed to overcome an egalitarian illiterateness trap is the same as the one described in the previous section. In contrast to this, we concentrate in this section on the case in which, without state intervention, the population remains split into two groups even in the long run. If the initial endowment of human capital is very unequally distributed, the economy might remain split into two groups in the long run even if the population is perfectly homogeneous in any aspect but initial endowments.

As in case of a homogeneous population, also in this case compulsory schooling is a necessary condition for the economy to overcome the illiterateness trap in which part of the population is stuck. For higher levels of compulsory schooling, subsidies and compulsory schooling are substitutes with respect to the ability to overcome the split of society.

Take, for example, the model with the following parameter values already extensively discussed in Section 3.2: $s = 0.5$, $\alpha = 0.3$, $\beta = \frac{2}{3}$, $\gamma = 0.5$, $\delta = 0.05$, and $\eta^A = \eta^B = \eta = 1$, implying that $h_c^A = h_c^B$. For these parameters we have already shown that without education policy there exists an inegalitarian steady state where one part of the population, say the *B*-types, is trapped in illiterateness. If the simulation starts in this inegalitarian steady state with illiterate *B*-types, the introduction of compulsory schooling will of course increase the human capital level of the *B*-types. For low levels of compulsory schooling, e.g. for $h_c^A = h_c^B \leq 1.06$, however, the *B*-types will never acquire more than the minimum level of human capital enforced by compulsory schooling. This result holds for any level of education subsidy. In other words, if there is too little compulsory schooling, the split of the society cannot be overcome.

For higher levels of compulsory schooling, education subsidies and compulsory schooling work as substitutes in their ability to overcome the inegalitarian steady state. Tables 2 and 3 depict the convergence levels of human capital and of

¹⁴ Details of the simulations are available from the authors upon request.

¹⁵ Recall that for the relevant education expenditures close to 0, the realized human capital level decreases in β .

Table 2Heterogeneous population with $h_c^A = h_c^B = 1.09$: $s = 0.5$, $\alpha = 0.3$, $\beta = \frac{2}{3}$, $\gamma = 0.5$, $\delta = 0.05$, and $\eta^A = \eta^B = 1$.

Subsidy	Steady state human capital		Utility	
	A	B	A	B
0	1.30437	1.09	-1.32372	-2.43744
0.1	1.35705	1.09	-0.979351	-2.53421
0.2	1.39955	1.09	-0.688271	-2.65155
0.3	1.43624	1.09	-0.443149	-2.79662
0.4	1.46817	1.09	-0.257243	-2.97814
0.45	1.34558	1.34558	-1.83506	-1.83506
0.5	1.35015	1.35015	-1.93195	-1.93195
0.6	1.3556	1.3556	-2.21079	-2.21079
0.7	1.35504	1.35504	-2.64805	-2.64805
0.8	1.34542	1.34542	-3.35404	-3.35404
0.875	1.49893	1.09	-2.18367	-5.28249
0.9	1.4837	1.09	-2.73252	-5.64131

Table 3Heterogeneous population with $h_c^A = h_c^B = 1.1$: $s = 0.5$, $\alpha = 0.3$, $\beta = \frac{2}{3}$, $\gamma = 0.5$, $\delta = 0.05$, and $\eta^A = \eta^B = 1$.

Subsidy	Steady state human capital		Utility	
	A	B	A	B
0	1.30177	1.1	-1.34614	-2.41029
0.1	1.3546	1.1	-1.00426	-2.69754
0.2	1.39709	1.1	-0.716696	-2.62396
0.3	1.43368	1.1	-0.475359	-2.76873
0.35	1.33306	1.33306	-1.70745	-1.70745
0.4	1.33988	1.33988	-1.76114	-1.76114
0.45	1.34558	1.34558	-1.83506	-1.83506
0.5	1.35015	1.35015	-1.93195	-1.93195
0.6	1.3556	1.3556	-2.21079	-2.21079
0.7	1.35504	1.35504	-2.64805	-2.64805
0.8	1.34542	1.34542	-3.35404	-3.35404
0.9	1.31726	1.31726	-4.66309	-4.66309
0.95	1.42461	1.1	-4.54689	-6.93195

utility levels of the two groups for different subsidy rates when the process starts in the inegalitarian steady state with illiterate B-types. Table 2 shows the results for $h_c^A = h_c^B = 1.09$, Table 3 for $h_c^A = h_c^B = 1.1$.

For both levels of compulsory schooling some education subsidies are necessary to overcome the split of society. The minimal subsidy rate necessary to reach an egalitarian steady state decreases in the level of compulsory schooling. It is 0.45 for $h_c^A = h_c^B = 1.09$ and 0.35 for $h_c^A = h_c^B = 1.1$. These results are qualitatively similar to the ones obtained with the same parameters when the whole population is stuck in illiterateness. The main difference is the amount of compulsory schooling and subsidies necessary to overcome a low human capital level. With a homogeneous population the system converges to the interior egalitarian steady state even for $h_c^A = h_c^B = 1.01$ when the subsidy rate is high enough. In contrast to this, in case of a heterogeneous population the split of society cannot be overcome with any subsidy rate whenever $h_c^A = h_c^B \leq 1.06$. This suggests that it is more difficult to overcome low human capital levels of a part of the population than of the whole population.

Furthermore, an egalitarian steady state needs not pareto-dominate an inegalitarian steady state. To see this compare the interior egalitarian steady state without policy intervention with the inegalitarian steady state without policy intervention favorable for the A-types.¹⁶ As mentioned in Section 3.2, without state intervention the utility of the favored A-types is -1.208 in the inegalitarian steady state, while the utility realized in the interior egalitarian steady state is -1.823 . So without government intervention the favored A-types are better off in the inegalitarian than in the egalitarian steady

¹⁶ Recall that even without government intervention an egalitarian interior steady state is realized if the initial endowments with human capital of both groups are not too small and not too different.

state. The reason for this is twofold. First, the overall level of human capital is lower in the inegalitarian steady state than in the egalitarian. This implies a higher wage rate per human capital unit in the inegalitarian steady state. Second, favored agents have a higher level of human capital in the inegalitarian steady state than in the egalitarian. Consequently, favored agents are better off in the inegalitarian steady state than in the egalitarian.

This discussion shows that the privileged part of the society might oppose education policies designed to overcome the inequality.¹⁷ If we compare, for example, the utility of the favored *A*-types in the inegalitarian steady state without education policy (i.e. -1.208) with the utility generated in the egalitarian steady arising from an education policy $h_c^A = h_c^B = 1.1$ and $\sigma = 0.35$ (i.e. -1.70745 , see Table 3), one sees that in the long run the *A*-types would loose from such a policy intervention. Some education subsidy is beneficial also for the favored types, however. For $h_c^A = h_c^B = 1.1$, the utility level of the *A*-types is increasing in σ as long as it is low enough not to induce convergence to an egalitarian steady state, i.e. as long as $\sigma < 0.35$ (see Table 3). The reason of this positive effect of subsidies is the same as in the economy with a homogeneous population—the subsidy helps to overcome the lack of parental internalization of children's welfare.

Education subsidies have two effects on the favored group. On the one hand, it helps the group by internalizing children's welfare. On the other hand, the privileged group looses if the society moves from an inegalitarian to an egalitarian steady state. Given these two opposing effects the question arises whether the positive first effect can ever be strong enough to compensate for the second effect. Comparing the utility levels of an egalitarian steady state with policy intervention as shown in Tables 2 and 3 suggests that the utility level of the *A*-types in any of the egalitarian steady states is lower than *A*'s utility in the inegalitarian steady state without policy intervention, i.e. -1.208 . We have done extensive simulations showing that for these parameter constellations it is impossible to introduce a split overcoming education policy from which also the *A*-types benefit in the long run.¹⁸

This conclusion does not hold if the share of favored types increases from 0.5 to 0.9. In this case the resulting inegalitarian steady state without policy intervention is such that the utility of the favored group is considerably lower, namely -1.734 instead of -1.208 . The reason for this drop in utility is straightforward—if there are more favored types, the overall endowment of the economy with human capital increases in the inegalitarian steady state, and hence the wage per human capital unit decreases. Consequently, the wage of the favored type is lower, translating into lower utility. With a high share of favored types, it is indeed possible to overcome the split of the society in such a way that also the previously favored type benefits in the long run. Take, e.g. an education policy of $h_c^A = h_c^B = 1.1$ and $\sigma = 0.35$. Such a policy induces an egalitarian steady state where the realized utility is -1.70745 (see Table 2), so also the favored types gain compared to an inegalitarian steady state without education policy.¹⁹

This discussion indicates an interesting, non-monotone relation between the share of the privileged and the political feasibility of an education policy designed to overcome the split of society. On the one hand, such an egalitarian policy with education subsidies should be feasible when the share of privileged is very large, because in this case also the privileged would benefit from such a policy. On the other hand, if the share of privileged is very small, a democratic political process should overcome the resistance of the few privileged. So it seems that in a democracy an egalitarian education policy is most difficult to implement when the size of both groups is not too different.

All the conclusions have to be modified when the split of society is not caused by different initial endowments, but by differences in talent. As we have seen in Section 3.2, differences in talent lead to a split in society even for the same initial distribution of endowments. Furthermore, except for very small talent differences, the less talented stay illiterate.

Take the model with the same parameters as in Section 3.2, i.e. $s = 0.5$, $\alpha = 0.3$, $\beta = \frac{2}{3}$, $\gamma = 0.5$, $\delta = 0.05$, and $\eta^B = 1$. When starting from the steady state without education policy, where the *B*-types are illiterate, compulsory schooling is necessary to overcome the illiterateness. Compulsory schooling and subsidies are again substitutes in their ability to overcome low human capital rates. For $h_c^B = 1.1$ and $h_c^A = 1.1\eta^A$ Table 4 shows how differences in talent, i.e. different η^A , affect the minimum subsidy needed to induce an interior steady state when the process starts at the steady state without education policy.

Without subsidies, even small talent differences, i.e. $\eta^A = 1.001$ are sufficient to induce the less talented *B*-types to do only compulsory schooling. The higher the talent difference, i.e. the higher η^A , the higher the education subsidy necessary to reach an interior steady state. However, note that for any level of talent difference the subsidy level optimal for the *A*-types is such that the *B*-types do only compulsory schooling. In other words, the more talented ones will never support a policy that induces a high level of education for the less talented ones. Hence, the political feasibility of such a policy remains doubtful.

¹⁷ A permanent split of the society might also be supported by the education system itself. In German speaking countries, pupils are selected into different school types at the age of ten. Studies show that this selection is mainly determined by the social background of the children and not by intelligence, talent, etc. Upper and upper middle class children are selected for 'good' schools allowing their graduates to attend university, whereas lower and lower middle class children are selected for schools oriented towards 'practical education'. Despite the obvious waste of talent associated with this system, any change of the system has been prevented by the political influence of those benefiting from the system. An analysis of such an effect is beyond the scope of this paper.

¹⁸ Detailed simulations results are available from the authors upon request.

¹⁹ Note that in an egalitarian steady state the share of *A*-types is irrelevant, since in this state both types are equal anyhow.

Table 4

Heterogeneous talents with education policy: $s = 0.5$, $\alpha = 0.3$, $\beta = \frac{2}{3}$, $\gamma = 0.5$, $\delta = 0.05$, and $\eta^B = 1$.

η^A	Min. subsidy for interior	Subsidy optimal for A	Subsidy optimal for B
1.001	0.35	0.325	0.35
1.01	0.375	0.35	0.375
1.02	0.4	0.375	0.4

5. Conclusions

We have shown that if parent's attitude towards the education of the children depends on their own education, the economy might get trapped in an illiterateness steady state where a low education level of the parents leads to the negligence of the children's education, reproducing the low education level in the next generation. To overcome such a steady state, compulsory schooling and education subsidies are necessary. Education subsidies can also mitigate the inefficiencies caused by the insufficient internalization of the children's welfare into parents' preferences.

Because of the intergenerational transmission of education attitudes the society might be split into different education groups even in the long run. This split can be overcome by an appropriate mixture of compulsory schooling and education subsidies. Also in this case the subsidy might help to reduce the inefficiencies caused by the insufficient internalization of the children's welfare into parents' preferences. However, such an egalitarian education policy might hurt the privileged group, reducing its political feasibility.

The model is based on the assumption that labor supply is fixed. Consequently, the taxation of wage income used to finance education subsidies does not create any excess burden on the labor market. If labor supply is elastic and if a non-distortive tax is not available, a trade-off exists between the inefficiency created by the tax system and the inefficiency due to the externalities in the human capital formation. The extent of the tax-induced excess burden depends of course on the elasticity of the labor supply, and its size is an empirical question which goes beyond the scope of this paper. The larger the inefficiency caused by taxation, the less attractive are education subsidies, and the more favorable is compulsory schooling as a mean to overcome low education levels. Obviously this conclusion holds only as long as compulsory schooling can be financed by the parents—a condition fulfilled for all the results reported above. If this condition fails, i.e. if the compulsory schooling necessary to overcome low education levels is unaffordable for low income parents, education subsidies are needed on top of compulsory schooling. Education subsidies might also be required for political reasons, i.e. in order to overcome the opposition of higher educated groups. So despite any excess burden created by taxation, public financing of schools and universities might be needed to overcome inefficiently low levels of human capital of (parts of) the population.

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Appendix A

Lemma 1. *If $\tilde{k}_t^A + \tilde{k}_t^B > 0$, FOCs (8) and (9) and the wage equation (4) fully characterize the competitive equilibrium.*

Proof. Note first that for finite values of $(\tilde{k}_t^I, \tilde{h}_t^I)$ the wage realized \tilde{w}_t^I has to be finite, implying that \tilde{h}_{t+1}^I and \tilde{k}_{t+1}^I have to be finite, too. The utility function is strictly quasiconcave, implying a unique solution, which might be either interior (in which case the first order conditions hold) or at the lower bounds. By (4) $\tilde{w}_t^I > 0$ whenever $\tilde{k}_t^A + \tilde{k}_t^B > 0$. Furthermore, $\tilde{w}_t^I - \tilde{k}_{t+1}^I = \tilde{c}_{2,t}^I + ((\tilde{h}_{t+1}^I/\eta^I) - 1)^{1/\beta} \geq \tilde{c}_{2,t}^I > 0$ due to the INADA condition of the utility function with respect to the consumption levels. This implies that $\partial U/\partial k_{t+1}^I = \infty$ at $k_{t+1}^I = 0$. This requires that the condition (9) as well as (ii) must hold.

As for the solution for the human capital, note first that for $\tilde{h}_t^I = \eta^B$, $\partial U/\partial h_{t+1}^I = 0$ at $h_{t+1}^I = \eta^B$. This gives (iii) and that condition (8) holds in this case.

If $\tilde{h}_t^I > \eta^B$, $\partial U/\partial h_{t+1}^I = \infty$ at $h_{t+1}^I = \eta^B$, implying $\tilde{h}_{t+1}^I > \eta^B$. This gives (iv) and that condition (8) holds also for $\tilde{h}_t^I > 1$, which completes the proof. \square

A.1. Proof of proposition 2

Using Lemma 1, insert the attitude function (10) and (4) into (8) and (9), setting $s = 0$ and $\eta^B = 1$, and dropping the superscripts denoting the type, we get

$$(1 - \alpha)\tilde{k}_t^\alpha \tilde{h}_t^{1-\alpha} = \left(\left(\frac{(1 + \gamma)}{\beta \frac{1}{\delta} (\tilde{h}_t - 1)} \right) \frac{\tilde{h}_{t+1}}{(\tilde{h}_{t+1} - 1)} + 1 \right) (\tilde{h}_{t+1} - 1)^{1/\beta}, \tag{14}$$

$$(1 - \alpha)\tilde{k}_t^\alpha \tilde{h}_t^{1-\alpha} = \left(\frac{(1 + \gamma)}{\gamma} + \frac{\beta \frac{1}{\delta} (\tilde{h}_t - 1)}{\gamma} \frac{(\tilde{h}_{t+1} - 1)}{\tilde{h}_{t+1}} \right) \tilde{k}_{t+1}. \tag{15}$$

Substituting (14) into (15) for k_t , ignoring the period indices, and rearranging terms leads to the following condition for an interior steady state:

$$(1 - \alpha) \left(\frac{\delta \gamma}{\beta} \right)^\alpha = \left(\frac{\delta(1 + \gamma)}{\beta} \right) (h^* - 1)^{((1/\beta)-2)(1-\alpha)} + \frac{1}{h^*} (h^* - 1)^{1/\beta - ((1/\beta)-2)\alpha}. \tag{16}$$

Define

$$lhs := (1 - \alpha) \left(\frac{\delta \gamma}{\beta} \right)^\alpha,$$

$$rhs(h^*) := \left(\frac{\delta(1 + \gamma)}{\beta} \right) (h^* - 1)^{((1/\beta)-2)(1-\alpha)} + \frac{1}{h^*} (h^* - 1)^{1/\beta - ((1/\beta)-2)\alpha},$$

$$a(h^*) := \left(\frac{\delta(1 + \gamma)}{\beta} \right) (h^* - 1)^{((1/\beta)-2)(1-\alpha)},$$

$$b(h^*) := \frac{1}{h^*} (h^* - 1)^{1/\beta - ((1/\beta)-2)\alpha}.$$

Proof of (i) lhs strictly positive. If $\beta < \frac{1}{2}$, $((1/\beta) - 2)(1 - \alpha) > 0$ and $1/\beta - ((1/\beta) - 2)\alpha > 2$. This implies that $\partial a/\partial h^* > 0$ and $\partial b/\partial h^* > 0$ — rhs is strictly increasing in h^* . Furthermore, $rhs(h^* = 1) = 0$, and $\lim_{h^* \rightarrow \infty} a(h^*) = \infty$. Therefore there exists exactly one $h^* > 1$ such that $lhs = rhs(h^*)$.

Proof of (ii) Again, lhs strictly positive. If $\frac{1}{2} < \beta < 1$, $((1/\beta) - 2)(1 - \alpha) < 0$ and $1 < 1/\beta - ((1/\beta) - 2)\alpha$. This implies that $\lim_{h^* \rightarrow 1} a(h^*) = \infty$, $\lim_{h^* \rightarrow \infty} a(h^*) = 0$, $\lim_{h^* \rightarrow \infty} b(h^*) = \infty$, and $b(h^* = 1) = 0$. This gives $\lim_{h^* \rightarrow 1} rhs(h^*) = \lim_{h^* \rightarrow \infty} rhs(h^*) = \infty$ and finite values of rhs for all other values of h^* .

Next we show that $rhs(h^*)$ has a unique local extremum in the interior. Because of $\lim_{h^* \rightarrow 1} rhs(h^*) = \lim_{h^* \rightarrow \infty} rhs(h^*) = \infty$, a unique interior local extremum must be a unique local minimum of $rhs(h^*)$. Uniqueness of the local minimum implies that $\partial rhs(h^*)/\partial h^* < 0$ for all values of h^* below this minimum and $\partial rhs(h^*)/\partial h^* > 0$ for all values of h^* above this minimum. In the interior, any local extremum is characterized by the condition

$$\frac{\partial rhs(h^*)}{\partial h^*} = 0$$

leading to

$$\frac{(h^* - 1)}{h^*} - \left(\frac{1}{\beta} - 2 \right) (1 - \alpha) \left(\frac{\delta(1 + \gamma)}{\beta} \right) \frac{h^*}{(h^* - 1)^2} = \frac{1}{\beta} - \left(\frac{1}{\beta} - 2 \right) \alpha. \tag{17}$$

For the left hand side of Eq. (17), we have

$$\lim_{h \rightarrow 1} \left(\frac{(h^* - 1)}{h^*} + z \frac{h^*}{(h^* - 1)^2} \right) = \infty,$$

with $z = -((1/\beta) - 2)(1 - \alpha)(\delta(1 + \gamma)/\beta) > 0$, and

$$\lim_{h \rightarrow \infty} \left(\frac{(h^* - 1)}{h^*} + z \frac{h^*}{(h^* - 1)^2} \right) = 1.$$

This implies that there is at least one local extremum in the interior. To check uniqueness, we will show $((h^* - 1)/h^*) + zh^*/(h^* - 1)^2$ is strictly decreasing in h^* for the relevant values of h^* . The first derivative of the left hand side is given by

$$\frac{\partial \left(\frac{(h^* - 1)}{h^*} + z \frac{h^*}{(h^* - 1)^2} \right)}{\partial h^*} = \frac{-zh^{*2}(h^* + 1) + (h^* - 1)^3}{h^{*2}(h^* - 1)^3}. \tag{18}$$

To see that this derivative is strictly negative for the relevant values of h^* , note first that $1/\beta - ((1/\beta) - 2)\alpha > 1$. All solutions to Eq. (17) must satisfy the condition

$$\frac{(h^* - 1)}{h^*} + z \frac{h^*}{(h^* - 1)^2} > 1,$$

which is equivalent to

$$zh^{*2} > h^*(h^* - 1)^2 - (h^* - 1)^3.$$

Inserting into (18) implies that

$$\frac{\partial \left(\frac{(h^* - 1)}{h^*} + z \frac{h^*}{(h^* - 1)^2} \right)}{\partial h^*} < \frac{-(h^*(h^* - 1)^2 - (h^* - 1)^3)(h^* + 1) + (h^* - 1)^3}{h^{*2}(h^* - 1)^3} = \frac{-2}{h^{*2}(h^* - 1)} < 0.$$

So the left hand side of (17) is strictly decreasing in the relevant area, and hence Eq. (17) has a unique solution. This implies that $rhs(h^*)$ has a unique local minimum in the interior whenever $\beta > \frac{1}{2}$, and that $\partial rhs(h^*)/\partial h^* < 0$ for all values of h^* below this minimum and $\partial rhs(h^*)/\partial h^* > 0$ for all values of h^* above this minimum. Furthermore, recall that $\lim_{h^* \rightarrow 1} rhs(h^*) = \lim_{h^* \rightarrow \infty} rhs(h^*) = \infty$. So for generic parameter values there are two possibilities: Either there exist two different h^* such that $lhs = rhs(h^*)$. In this case there are two interior steady states. On the other hand, it is possible that $lhs < rhs(h^*)$ for all $h^* > 1$ which implies that there is no interior steady state.

By example we show that both possibilities are indeed feasible. Take first the case $\alpha = 0.3, \beta = 0.7, \gamma = 0.5, \delta = 0.01$ and then the case. $\alpha = 0.3, \beta = 0.7, \gamma = 0.5, \delta = 0.04$. In the first case condition (16) can be written as

$$0 = (1 - 0.3) \left(\frac{(0.1)(0.5)}{0.7} \right)^{0.3} - \left(\frac{(0.1)(1 + 0.5)}{0.7} \right) (h^* - 1)^{(1/0.7) - 2(1 - 0.3)} - \frac{1}{h^*} (h^* - 1)^{1/0.7 - ((1/0.7) - 2)0.3} = F(h^*)$$

and in the second case it can be written as

$$0 = (1 - 0.3) \left(\frac{(0.04)(0.5)}{0.7} \right)^{0.3} - \left(\frac{(0.04)(1 + 0.5)}{0.7} \right) (h^* - 1)^{(1/0.7) - 2(1 - 0.3)} - \frac{1}{h^*} (h^* - 1)^{1/0.7 - ((1/0.7) - 2)0.3} = G(h^*).$$

When trying to solve $F(h^*) = 0$ for h^* one finds no solution, whereas solving $G(h^*) = 0$ gives exactly two solutions: $h_1^* = 1.09496$ and $h_2^* = 1.2675$.

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