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# LEARNING TO LIKE WHAT YOU HAVE – EXPLAINING THE ENDOWMENT EFFECT

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## Appendix

*Proof of Lemma* 1 To prove uniqueness suppose there exist two Nash bargaining solutions $(x, y)$ and $(x', y')$. Since utility functions are strictly concave, both individuals would prefer any convex combination of $(x, y)$ and $(x', y')$ over $(x,y)$ and $(x', y')$. These convex combinations are feasible which yields a contradiction.

*Proof of Lemma* 2 If Pareto improving allocations exist, the maximised Nash product $N(x^*, y^*)$ must be strictly positive, i.e. both individuals must be strictly better off than with their endowment. Such a Pareto improving allocation exists if the problem

$$\max_{x,y} U_2(x,y) \qquad \text{s.t.} \qquad F(x,y) + e_1 x = e_1$$

has a value $U_2 > e_2$. Let $\tilde{x}(y)$ denote the $x$ that solves the constraint for a given $y$. Implicitly differentiating we find that[1]

$$\frac{\mathrm{d}\tilde{x}}{\mathrm{d}y} = \frac{-F_y}{F_x + e_1}.$$

Substituting $\tilde{x}(y)$ into $U_2(x, y)$ and differentiating yields

$$\frac{\mathrm{d}U_2}{\mathrm{d}y} = -F_{2x}\frac{\mathrm{d}\tilde{x}}{\mathrm{d}y} - F_{2y} - e_2.$$

In particular, a Pareto improving allocation exists if

$$\left.\frac{\mathrm{d}U_2}{\mathrm{d}y}\right|_{y=0} > 0$$

or

$$F_{2x}(1,0)\frac{F_y(1,0)}{F_x(1,0) + e_1} - F_{2y}(1,0) - e_2 > 0.$$

Noting that $\lim_{y\to 0} F_y(1, y) = +\infty$ and $\lim_{x\to 1} F_{2x}(x, 0) = \lim_{x\to 0} F_x(x, 1) = +\infty$ implies that this condition is always fulfilled. Hence, $U(x^*, y^*) - e_1 > 0$ and $U_2(x^*, y^*) - e_2 > 0$. □

*Proof of Lemma 3* At any boundary solution one of the individuals, say individual 1, receives nothing of one of the goods. Let us first look at an allocation with $y = 0$. In this case individual 1 is no better off than with his initial endowment, and hence such an allocation

[1] Recall that we have assumed that $I$ is bounded below such that the goods always remain ‘goods’. Here, this implies $F_x + e_1 > 0$.

cannot be a bargaining solution by Lemma 2. Now turn to an allocation with $x = 0$. For this to be a bargaining solution it must hold that

$$\frac{\partial N(x,y)}{\partial x} = [F_x(0,y) + e_1][U_2(0,y) - e_2] - [U(0,y) - e_1]F_{2x}(0,y) \leq 0.$$

But since $U_2(x^*, y^*)-e_2>0$ and $\lim_{x\to 0} F_x(x, y) = +\infty$, whereas $F_{2x}(0, y) = F_x(1, 1-y)$ is finite, this condition can never be fulfilled. Hence, there cannot be a boundary solution where individual 1 gets nothing. By symmetry, also all boundary solutions with individual 2 getting nothing of one of the goods can not be a solution. Hence, the solution must always be in the interior.

*Proof of Proposition* 1 By Lemmata 2 and 3 for all $e_1$, $e_2$ the solution is interior, and the constraints $U(x, y) \geq e_1$ and $U_2(x, y) \geq e_2$ are not binding. Hence, $x^*$ and $y^*$ are simultaneously determined by the first order conditions, $N_x = 0$, $N_y = 0$. By the implicit function theorem $x^*(\cdot,\cdot)$ and $y^*(\cdot,\cdot)$ are differentiable in $e_1$ and $e_2$.[2]

We will show that for all $e_1 \leq 0$

$$\frac{\partial F[x^*(e_1,e_2), y^*(e_1,e_2)]}{\partial e_1} = F_x\frac{\partial x^*}{\partial e_1} + F_y\frac{\partial y^*}{\partial e_1} > 0. \tag{2}$$

Differentiating $N_x = 0$, $N_y = 0$ with respect to $e_1$, we get that

$$\frac{\partial x^*}{\partial e_1} = \frac{N_{ye_1}N_{xy} - N_{xe_1}N_{yy}}{N_{xx}N_{yy} - (N_{xy})^2} \text{ and } \frac{\partial y^*}{\partial e_1} = \frac{N_{xe_1}N_{xy} - N_{ye_1}N_{xx}}{N_{xx}N_{yy} - (N_{xy})^2}. \tag{3}$$

Since the second order necessary condition, $N_{xx}N_{yy}-(N_{xy})^2 \geq 0$, is satisfied at an interior solution, we will show that at $e_1 \leq 0$

$$N_{xe_1}(F_yN_{xy} - F_xN_{yy}) + N_{ye_1}(F_xN_{xy} - F_yN_{xx}) > 0.$$

Since

$$\begin{aligned} N_{xe_1} &= F_2 - e_2y + F_{2x}(1-x) > 0 \\ N_{ye_1} &= (1-x)(F_{2y} + e_2) \geq 0, \end{aligned}$$

it holds that

$$\begin{aligned} N_{xe_1}\left[-2F_ye_1(F_{2y} + e_2)\right] + N_{ye_1}2F_yF_{2x}e_1 = \\ -2(F_2 - e_2y)e_1F_y(F_{2y} + e_2) \geq 0. \end{aligned}$$

Note that $F_2 > e_2y$. Otherwise the allocation would not be Pareto improving. Thus, it suffices to show that

$$F_yN_{xy} - F_xN_{yy} > -2F_ye_1(F_{2y} + e_2) \tag{4}$$

and

$$F_xN_{xy} - F_yN_{xx} > 2F_yF_{2x}e_1. \tag{5}$$

At an interior solution $MRS_1 = MRS_2$, which implies that

[2] Strictly speaking, the implicit function theorem requires the second order condition $N_{xx}\,N_{yy}-(N_{xy})^2$ to be non–zero. We assume this to be satisfied.

$$\frac{F_x + e_1}{F_y} = \frac{F_{2x}}{F_{2y} + e_2}.$$

Thus, we have

$$F_y F_{2x} = F_x(F_{2y} + e_2) + e_1(F_{2y} + e_2) \tag{6}$$

Given that

$$\begin{aligned} N_{xy} = & F_{xy}(F_2 - e_2 y) + F_{2xy}(F + e_1 x - e_1) \\ & - F_x(e_2 + F_{2y}) - F_y F_{2x} - e_1(F_{2y} + e_2) \end{aligned}$$

we can define

$$N' := -2F_y F_{2x} < N_{xy},$$

where the inequality follows from (6). To verify claim (4) we show that $F_y N' - F_x N_{yy} > -2F_y e_1(F_{2y} + e_2)$ at $e_1 \leq 0$.

$$\begin{aligned} & F_y N' - F_x N_{yy} \\ & = 2F_x F_y e_2 + 2F_y(F_x F_{2y} - F_y F_{2x}) \underbrace{-F_x F_{yy}(F_2 - e_2 y)}_{>0} \underbrace{-F_x F_{2yy} F}_{>0}. \end{aligned}$$

By (6) it follows that

$$2F_x F_y e_2 + 2F_y(F_x F_{2y} - F_y F_{2x}) = -2F_y e_1(F_{2y} + e_2),$$

which proves (4).

Equation (5) is satisfied if $F_x N' - F_y N_{xx} > 2F_y F_{2x} e_1$. But this follows immediately:

$$\begin{aligned} & F_x N' - F_y N_{xx} \\ & = -2F_x F_y F_{2x} - F_y(F_{2xx} F - 2(F_x + e_1)F_{2x} + F_{xx}(F_2 - e_2 y)) \\ & = 2F_y F_{2x} e_1 \underbrace{-F_y F_{2xx} F}_{>0} \underbrace{-F_y F_{xx}(F_2 + e_2 y)}_{>0}. \end{aligned}$$

Therefore, fitness is strictly increasing in $e_1$ for all $e_1 \leq 0$. □