Market Evolution

1. Introduction

Trading requires an institutional framework that determines the matching, the information, and the price formation process.

Huge variety of market institutions exists in the field

But: Market equilibrium concepts do not take the institution into account

Empirical and experimental evidence: Instituions matter for realized prices, quantities etc $\Longrightarrow$

How do market institutions evolve?

What are the driving forces behind the evolution of market institutions?

Is there any mechanism that guarantees that existing market institutions support market-clearing outcomes?

Is there any mechanism that guarantees that actual markets are characterized by efficient institutions?

Two aspects of the evolution of trading platforms

The emergence of new institutions

Survival of (i.e. competition between) existing trading institutions

Claims

Because of efficiency reasons, only trading institutions that guarantee market clearing survive in the long run.

In the long run, traders learn to use market-clearing institutions.

Question: If traders have to choose between different trading institutions, will they learn to choose a market-clearing one?

2. The Model

2.1. Traders

one homogenous good (partial equilibrium model)

n identical buyers with demand function $d(p)$; m identical sellers with supply function $s(p)$ (identical only for ease of exposition)

$$\begin{aligned} d'(p) &< 0;\ s'(p) > 0 \\ d(0) &> 0;\ s(0) = 0 \\ \lim_{p\to\infty} d(p) &= 0 \end{aligned}$$

rationing possible $\Longrightarrow$ market outcomes (p, q_B), (p, q_S)

evaluation of the outcome - payoff-functions

$$v_B(p, q_B);\ v_S(p, q_S)$$

Note: our framework more general than standard demand/supply derived from utility/profit maximization.

$A1$: It holds for all for all p, p' with $0 < p < p'$ and $d(p) > 0$ that

$$\begin{aligned} v_B(p, d(p)) &> v_B(p', d(p')) \\ v_S(p, s(p)) &< v_S(p', s'(p)) \end{aligned}$$

In absence of rationing, a lower price is better for the buyer and worse for the seller.

$A2$: It holds for all for all p and all $q_B < d(p)$, $q_S < s(p)$ that

$$\begin{aligned} v_B(p, d(p)) &> v_B(p, q_B) \\ v_S(p, s(p)) &> v_S(p, q_S) \end{aligned}$$

Given the price, traders prefer not to be rationed.

$A3$: It holds for all for all p, p' and all q_B, q_S with $0 < q_B < d(p)$, $0 < q_S < s(p)$ that

$$\begin{aligned} v_B(p, q_B) &> v_B(p', 0) \\ v_S(p, q_S) &> v_S(p', 0) \end{aligned}$$

Traders prefer rationed trade at a price with strictly positive demand and supply, respectively, over not being able to trade.

$A1$-$A3$ fulfilled by standard model

2.2.Trading Institutions

good traded at different institutions

traders have to choose the institution at which they want to trade

m_z and n_z denote numbers of sellers and buyers who have chosen institution z

market clearing price $p^*(m_z, n_z)$ of z is given by solution of

$$m_z s(p) = n_z d(p)$$

institution z characterized by bias β_z

actual realized price at z

$$p_z(m_z, n_z, \beta_z) = \beta_z p^*(m_z, n_z)$$

$\beta_z = 1$: market-clearing institution

$\beta_z \neq 1$: non-market clearing institution - all traders at the long market side equally rationed:

$\beta_z > 1$: $q_{zB} = d(p_z(m_z, n_z, \beta_z))$; $q_{zS} = \frac{n_z}{m_z} d(p_z(m_z, n_z, \beta_z))$

$\beta_z < 1$: $q_{zB} = \frac{m_z}{n_z} s(p_z(m_z, n_z, \beta_z))$; $q_{zS} = s(p_z(m_z, n_z, \beta_z))$

$V_B(m_z, n_z, \beta_z)$, $V_S(m_z, n_z, \beta_z)$: buyer's and seller's payoff realized on z, if m_z, n_z sellers and buyers, respectively, have chosen z.

$A4$: For any fixed m_z, n_z with $m_z > 0$, $n_z > 0$, there exist a $\underline{\beta}(m_z, n_z) < 1 < \overline{\beta}(m_z, n_z)$ such that

$$\begin{aligned} V_B(m_z, n_z, \beta_z) &> V_B(m_z, n_z, 1) \text{ for all } \beta_z \in (\underline{\beta}(m_z, n_z), 1) \\ V_S(m_z, n_z, \beta_z) &> V_S(m_z, n_z, 1) \text{ for all } \beta_z \in (1, \overline{\beta}(m_z, n_z)) \end{aligned}$$

Lemma 1: Consider any distribution of traders where both a market-clearing institution 0 and a non-market clearing institution z are active. Under $A1$, $A2$ it hold that

$$v_B(p_z q_{zB}) \geq v_B(p_0 q_{0B},) \implies v_S(p_0 q_{0S}) > v_S(p_z q_{zS}).$$

Definition: A non-market clearing institution z is favored, if

$$v_B(p_0 q_{0B},) \geq v_B(p_z q_{zB}) \implies v_S(p_z q_{zS}) > v_S(p_0 q_{0S}).$$

Lemma 2: If $A1$ and $A4$ holds, then for a given number of buyers and sellers there exist favored institutions with a β that is in an open neighbourhood of 1.

Note: set of favored institutions might depend on n and m.

2.3. The Choice of the Institution

Players choose simultaneously among a finite set of feasible institutions (market clearing institution feasible).

Trades are conducted and outcomes evaluated (payoffs derived).

Coordination game - Due to $A3$, full coordination at any institution is a strict Nash-equilibrium, even coordination at institution leading to pareto-inferior outcome.

Do traders learn to coordinate on market clearing institution?

2.4. The Learning Model

At the end of a period t, traders observe outcome (prices and quantities) of all institutions active at t.

If a trader is allowed to revise his choice of institution, he switches to the institution with the outcome at t, which is best for him.

$\implies$ Markov process, where state ω given by a distribution of traders over institutions; state in period t, ω_t, determines probabilities with which each state is reached in next period $t+1$.

given ω_{t+1}, trade is conducted, outcomes are evaluated, and learning takes place determining ω_{t+2}, etc.

Note: since traders are homogenous, this learning behavior is equivalent to imitation learning. But homogeneity assumed only for ease of exposition.

2.5. Insert - Markov Processes

finite set of all possible states $\Omega = \{\omega_1, \omega_2, \omega_2, ...\}$

definition of the state depends on the learning

In our case: A state describes the distruibution of traders over the feasible trading platforms

Transition matrix (or Markov chain) P: element p_{ij} gives probability that state i is followed by state j in next period.

In our case: probability with which a trader switches to a particular platform (or stays) is derived from the probability that he is allowed to switch (see below) and from the trading outcomes of all active platforms. p_{ij} derived from the indivdual probabilities of switching.

Experimentation (mistakes): On top of the learning process characterized by P, every player experiments (makes mistakes) with a common probability ϵ. In case of experimentation a player chooses the trading platform according to some pre-specified full support probability distribution.

$\lim_{\epsilon \to o} P(\epsilon) = P$

Notation

Take any Markov chain M. A positive probability path from state ω_1 to state ω_k is a finite chain of states $(\omega_1, \omega_2, \omega_3, ...\omega_k)$ such that $m_{i,i+1} > 0$ for all $i = 1, 2..k - 1$.

A Markov chain M is irreducible, if there exists a positive probability chain from any state i to any state j

Obviously, the Markov process $P(\epsilon)$ is irreducible for $\epsilon > 0$, even if $p_{i,j} = 0$ for some states i, j.

Long Run Predictions

Definition: A nonempty set of states, A, is absorbing with respect to a Markov chain M, iff
i) $m_{i,j} = 0$ for all $\omega_i \in A$, $\omega_j \notin A$
ii) for all $\omega_i \in A$ there exists a positive probability path from state ω_i to any other state state $\omega_k \in A$

"A is absorbing, if it is a minimal subset of states which, once entered, is never abandoned."

If M is irreducible, the only absorbing set is Ω.

From now on: absorbing set are always meant to be absorbing sets of Markov learning P without experimentation.

Definition: For any Markov chain M a probability distribution over states, $\mu \in \Delta(\Omega)$, is an invariant distribution if $\mu \cdot M = \mu$.

Result: For every absorbing set A of a Markov chain M there is an unique invariant distribution, which has support A.

$\Longrightarrow$ For any $\epsilon > 0$, there is an unique invariant distribution $\mu(P(\epsilon))$.

From now on: invariant distribution always refers to the Markov chain with experimentation, $P(\epsilon)$.

Result: If M is irreducible, then for any state ω_i, the probability $\mu(\omega_i)$ of the unique invariant distribution gives the portion of periods, in which the process is at ω_i in the long run.

Definition: The limit invariant distribution μ^* is given by

$$\mu^* = \lim_{\epsilon \to o} \mu(P(\epsilon))$$

Definition: A state ω is stochastically stable, if it is in the support of μ^*.

Because of previous result: A state, which is not stochastically stable, is in the long run observed only in a negligible portion of periods, if experimentation becomes rare.

"Stochastically stable states do not die out in the long run."

How to characterize stochastically stable states?

Result: Only states which belong to an absorbing set of the dynamics without experimentation can be stochastically stable.

Definition: Given two absorbing sets A and B, let $c(A, B) > 0$ (referred to as the transition cost from A to B) denote the minimal number of experiments necessary for a positive probability path from an element of A into an element of B.

Definition: Let A be an absorbing set.

i) The radius of A is given by $R(A) = min\{c(A, B)|B$ is an absorbing set, $B \neq A\}$.

ii) The Coradius of A is given by $CR(A) = max\{c(B, A)|B$ is an absorbing set, $B \neq A\}$

"The Radius measures the minimal transition costs to get out of A into another absorbing set B. The Coradius measure the maximum transition costs to get from any absorbing set B to A."

Theorem: (Ellison 2000)

(i) If $R(A) \geq CR(A)$, all the states in A are stochastically stable.

(ii) If $R(A) > CR(A)$, the only stochastically stable states are those in A.

(iii) If the states in an absorbing set B are stochastically stable and $R(A) = c(B, A)$, the states in A are also stochastically stable.

Theorem allows for many games with stochastic learning models implying a Markov process a simple way to find stochastically stable states.

But: The conditions on radius and coradius are sufficient, but not always necessary - in general not all stochastically stable states are characterized.

2.6. The Model continued

Random revision opportunity

$E(k,\omega)$: event, that trader k receives revision opportunity in state ω.

$E^*(k,\omega)$: event, that k is the only agent of his type with revision opportunity in state ω.

$D1$: $Pr(E^*(k,\omega)) > 0$ for every agent k and state ω.

$D2$: For every agent k and state ω, either $Pr(E^*(k,\omega) \cap E(k',\omega)) > 0$ for any agent k' of the other market side, or $Pr(E^*(k,\omega) \cap E(k',\omega)) = 0$ for any such k'.

This general framework encompasses many standard learning models, like those with

independent inertia: Exogenous, independent, strictly positive probability, that an agent does not revise.

non-simultaneous learning: only one agent per period has positive probability of revision.

experimentation probability $\epsilon > 0$

in case of experimentation: institution chosen at random, with prob. distribution with full support over institutions

$\Longrightarrow$ unique invariant distribution $\mu(\epsilon)$ over the states with full support

limit invariant distribution $\mu^* = \lim_{\epsilon \to 0} \mu(\epsilon)$

2.7. Stochastically Stable Institutions

stochastically stable states are states in the support of μ^*.

Lemma 3: Under $A1$-$A3$, $D1$ and $D2$, only states with full coordination on one institution are stochastically stable.

stochastically stable institutions: institutions at which traders coordinate in stochastically stable states.

Theorem 1: Under $A1$-$A3$, $D1$ and $D2$, the market clearing institution is stochastically stable.

Theorem 2: Under $A1$-$A4$, $D1$ and $D2$, any of the favored institutions is stochastically stable.

Set of favored institutions might degenerate, if market size increases.

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2.8. Stable institutions and the market size

k-replica market: kn buyers, km sellers, $k \in \mathbb{N}$

Problem with asummptions $D1$ and $D2$ when k becomes large: Portion of traders allowed to switch might converge to zero $\Rightarrow$

revision probabilities: $\text{prob}_k(k \text{ buyer revise}) > 0$; $\text{prob}_k(k \text{ seller revise}) > 0$

Theorem 3:
If $n \geq m$, all favored institutions z of the original market with $\beta_z < 1$ are stochstically stable for the k-replica market when k is large enough.
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3. Conclusion

It is not excluded that traders learn to coordinate on a market clearing institution, but it is not guaranteed neither - they might as well coordinate on another, non-market clearing institution.