Design of Matching Markets

1. Introduction

Since 1990, economists are involved in the design of markets, e.g. matching markets.

Entry-level labor market for US-doctors

Hospitals hire entry level doctors for residency (internship)

Crucial for hospitals and for future career of doctors

Problems with this markets since 1940s

1940: doctors hired two years before graduation

Since 1951: Clearing house

Worked well until 1980's. Then

Legitimacy problems: Clearing house perceived to favor hospitals, applicants recommended to game system (not reveal true preferences)

Problem of spouses

2. A Simple Matching Model

set of workers (applicants) $W = \{w_1, ...w_n\}$, each searching for one job

set of firms (hospitals) $F = \{f_1, ...f_p\}$, each having q_f jobs to fill

matching: subset of $F \times W$, i.e. a set of matched pairs, such that any workers appears in at most one pair, and any firm f_i at most in q_i pairs

matching characterized by correspondence

$$\mu : F \cup W \rightarrow F \cup W$$

with $\mu(w_k) = \{f_j\}$ and $w_k \in \mu(f_j)$ iff w_k and f_j are matched.

if w_k is not matched, then $\mu(w_k) = \{w_k\}$

Each agent k has a complete and transitive preference ordering P_k over *acceptable* agents of the other market side, i.e. all those agents k prefers to be matched with over the outside option.

Problem: firms preferences defined over group of workers not individual workers

Preferences over individual workers can be derived from preferences over groups of workers, if latter are "responsive"

Responsive preferences of firm k: Take any two acceptable workers w and w', and any two subsets of acceptable workers S and S' that do not fill all the positions of k and that do not contain w and w'. It holds:

$$\left(S \cup \{w\} \succ_k S \cup \{w'\}\right) \quad \Leftrightarrow \quad \left(S' \cup \{w\} \succ_k S' \cup \{w'\}\right)$$
$$S \cup \{w\} \quad \succ \quad _k \quad S$$

"Responsive preferences allow for unambiguous projection of the preferences on set of individual workers" blocked matchings:

A matching μ is blocked by agent k if $\mu(k)$ contains an agent unacceptable for k agent

A matching μ is blocked by a pair of agents (f, w) iff

$$\begin{aligned} \text{i) } f &\succ_w \mu(w) \\ \text{ii) } w &\succ_f w' \text{ for some } w' \in \mu(f), \text{ or} \\ |\mu(f)| &< q_f \text{ and } w \text{ is acceptable to } f. \end{aligned}$$

Stability: A matching is stable, if it is not blocked by any individual or any pair of agents.

If a clearing mechanism does not produce stable matchings, agents have incentive to circumvent the clearing mechanism

Empirical and experimental evidence: Mechanism producing unstable matchings collapse

There always exists a stable matching, derived by the Deferred Acceptance Algorithm (DFM Gale, Shapley 1962)

Deferred Acceptance Algorithm with proposing workers (DFMW):

Step 1a: Each worker applies to his first choice firm.

Step 1b: Each firm f rejects any unacceptable application. If it receives more than q_f acceptable applications, it "holds" the q_f best ones

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Step ka: Any worker whose application was rejected at step $k-1$ makes an offer to the best acceptable firm that has not rejected him yet.

Step kb: Each firm f rejects any unacceptable application. It holds the q_f best acceptable applications from those it already held in step $k-1$ and the new ones.

DFM stops when no further applications are made. Each firm is matched with all those applicants it "holds" when DFMW stops.

Resulting matching μ_w

μ_w is stable.

Obviously, no offer is made to a firm unacceptable to the worker, and no firm holds an unacceptable worker $\Rightarrow$ no individual can block

Suppose there is a blocking pair (w, f). Then $f \succ_w \mu_w(w)$. Then w must have applied to f before the final step of the algorithm, and rejected by f. Hence, f does not prefer w to any of his workers $(\mu_w(w))$, and (w, f) is not blocking.

Theorem 1: The set of stable matchings is never empty.

Theorem 2: There exists no other stable matching where any worker is better of than with μ_w. The parallel firm proposing mechanism DFMF results in a matching μ_f that is the best stable matching for firms. The best stable matching for one side is the least preferred stable matching for the other side.

Theorem 3: At every stable matching, the same workers get a job (at different firms), and the same positions are filled (but by different workers) - "Outsiders never become insiders."

Theorem 4: With DFMW it is a dominant strategy for workers to state their true preferences (i.e. make their applications in the order of their true preferences). With DFMF it is a dominant strategy for firms to state their true preferences (i.e. to make offers to their most preferred workers). There exists no mechanism producing a stable matching such that it is a dominant strategy for all agents to state their true preferences.

1951 clearinghouse for doctors:

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interviews to determine acceptable candidates and hospitals, respectivly

each hospital and each candidate submitted preferences over acceptable matches

clearing with a mechanism equivalent to DFMF $\Rightarrow$

matchings stable
unfavorable for workers
workers have incentive not to state true preferences

Additional problem: spouses $\Rightarrow$ couples have preferences over pairs of positions

3. Matching with Couples

Set of applicants $A = A_1 \cup C$

A_1 set of singles, preferences over acceptable firms, i.e. subset of F

C set of couples $\Rightarrow$

A_2 set of wifes, A_3 set of husbands

Set of individual applicants $A' = A_1 \cup A_2 \cup A_3$

each couple c has a single preference over acceptable ordered pairs of positions, i.e. over subset of $F \times F$

each firm has preferences over acceptable individual applicants (i.e. subset of A')

as before matching is a set of pairs in $F \times A'$

Stability:

blocked matchings:

A matching μ is blocked by a firm or a single applicant k if $\mu(k)$ contains an agent unacceptable for agent k

A matching μ is blocked by a couple if they are matched to an unacceptable pair of positions

A single applicant w and a firm f block a matching if

$$\begin{aligned} \text{i) } f &\succ_w \mu(w) \\ \text{ii) } w &\succ_f w' \text{ for some } w' \in \mu(f), \text{ or} \\ |\mu(f)| &< q_f \text{ and } w \text{ is acceptable to } f. \end{aligned}$$

Two firms f_1 and f_2 and a couple c block a matching, if $(f_1, f_2) \succ_c \mu(c)$, and if either each of the firms prefers the corresponding part of the couple, or if one firm prefers "its" part of the couple, and the other firm hires the other part according to the matching, anyhow.

A matching is stable, when it cannot be blocked

The set of stable matchings might be empty (Roth 1984), and hence no algorithm guarantees convergence to a stable matching

Reason: couples have preferences over pairs of positions. In particular $(f, g) \succ_c (f', g')$ is consistent with $(f', g') \succ_c (f', g)$.

Mechanism used since 1995 for doctoral job: As before

hospitals, singles and couples submit preferences over acceptable "partners" (or pair of partners)

applicant (single or partner) is chosen, and matched with best firm(s) that accept him/them

next applicant is chosen, and matched with best accepting firm(s)

etc.

But: in this process, a one part of a couple previously matched might no longer be "hold" by her firm f.

f put on the "hospital stack", to be checked later

At the end of the round, check for blocking pairs (which have to involve hospitals from the hospital stack)

If blocking pairs or couples, one of blocking applicants chosen, and he/they can are matched with the best accepting firm(s)

etc.

Nothing guarantees that this process converges and the Theorems hold

But: Computational experiments with "real" data

Always convergence to stable matching

Applicant and firm proposer mechanism produce very similar results - no real incentive to play the system

$\Rightarrow$ mechanism without any problems