

GRADUATE MICROECONOMICS I

PROBLEM SET 7

Fall 2013

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1. Consider a game Γ_N , with players 1, 2 and 3 in which $S_1 = \{L, M, R\}$, $S_2 = \{U, D\}$ and $S_3 = \{l, r\}$. Player 1's payoffs from each of his three strategies conditional on the strategy choices of players 2 and 3 are depicted as (u_L, u_M, u_R) in each of the four boxes shown below, where $(\pi, \epsilon, \eta) \gg 0$. Assume $\eta < 4\epsilon$.

		Player 3	
		l	r
Player 2	U	$\pi + 4\epsilon, \pi - \eta, \pi - 4\epsilon$	$\pi - 4\epsilon, \pi + \eta/2, \pi + 4\epsilon$
	D	$\pi + 4\epsilon, \pi + \eta/2, \pi - 4\epsilon$	$\pi - 4\epsilon, \pi - \eta, \pi + 4\epsilon$

- (a) Argue that (pure) strategy M is never best response for player 1 to any independent randomization by players 2 and 3.
- (b) Show that (pure) strategy M is not strictly dominated.
- (c) Show that (pure) strategy M can be best response of player's 2 and player 3's randomizations are allowed to be correlated.
2. Consider the following normal form game:

		Player 2		
		L	C	R
Player 1	U	$1,0$	$6,\eta$	$0,4$
	M	$2,4$	$3,3$	$1,1$
	D	$5,1$	$-1,0$	$\epsilon,3$

- (a) Can one find conditions on ϵ, η so that the iterative elimination of strictly dominated strategies reduces the set of strategies to singletons for both agents?
- (b) Can one find conditions on ϵ, η so that the iterative elimination of not best response strategies reduces the set of strategies to singletons for both agents?

(c) if $\eta = 1$ and $\epsilon = -1$, what is the set of pure Nash equilibria?

3. Consider the following normal form game:

		Player 2	
		Left	Right
Player 1	Top	3,1	0,0
	Down	0,0	1,3

(a) Determine all the pure strategy Nash Equilibria of this game.

(b) Determine the mixed strategy equilibria.

4. Consider the following strategic form representation of a simultaneous move game:

		Player 2		
		<i>L</i>	<i>C</i>	<i>R</i>
Player 1	<i>U</i>	0,2	3,1	4,3
	<i>M</i>	2,4	0,3	3,2
	<i>D</i>	1,1	2,0	2,1

(a) Is there a dominant strategy for either of the two agents?

(b) Which strategies can always be eliminated because they are dominated?

(c) Which strategies can be eliminated if it is common knowledge that both players are rational?

(d) What are are Nash equilibria in pure strategies?

5. Consider the following strategic form representation of a simultaneous move game:

		Player 2		
		<i>L</i>	<i>C</i>	<i>R</i>
Player 1	<i>U</i>	0,2	2,0	3,1
	<i>M</i>	2,0	0,2	3,1
	<i>D</i>	1,3	1,3	4,4

(a) Identify the best response for each of the players.

(b) What are are Nash equilibria in pure strategies?

6. A taxpayer has income y that should be reported in full to the tax authority. There is a flat (proportional) tax rate γ on income. The reporting technology means that the tax payer must report income in full or zero income. The tax authority can choose whether or not to audit the taxpayer. Each audit costs an amount ϕ and if the audit uncovers under-reporting then the taxpayer is required to pay the full amount of tax owed plus a fine F .
- Set the problem out as a game in strategic form where each agent (taxpayer, tax authority) has two strategies.
 - Explain why there is no simultaneous move in pure strategies.
 - Find the mixed-strategy equilibria.
7. Consider two individuals, $i = A, B$, aiming at financing a public project. Each contributor i has wealth $w_i > 1/4$ and is free to contribute the sum c_i to the public project, with $0 \leq c_i \leq w_i$. The contributed money is used to produce a public good y according to the production function:

$$y = \sqrt{c_1 + c_2}$$

Each contributor i 's preference relation is represented by the utility function:

$$u_i(c_1, c_2) = y + w_i - c_i$$

Find all pure-strategy Nash equilibria.

8. Players 1 and 2 are bargaining over how to split one dollar. Both players simultaneously name shares they would like to have, s_1 and s_2 ; where $s_1, s_2 \in [0, 1]$. If $s_1 + s_2 \leq 1$, then the players receive the shares they named. If $s_1 + s_2 > 1$, then both players receive zero. What are the pure-strategy Nash equilibria of the game?

Students are also encouraged to the following exercises:

- MWG 8.B.1
- MWG 8.B.7

3. MWG 8.D.2

4. MWG 8.D.4

5. MWG 8.D.9