# Graduate Microeconomics I
# Problem Set 4

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1. There are three goods. Goods 1 and 2 are inputs. The third, with amount $q$, is the output. Output can be produced by two techniques that can be operated simultaneously or separately. The techniques are not necessarily linear. The first (respectively, second) technique uses only the first (respectively, second) input. Thus the first (respectively, the second) technique is completely specified by $\phi_1(q_1)$ [respectively, $\phi_2(q_2)$], the minimal amount of input one (respectively, two) *sufficient* to produce the amount of output $q_1$ (respectively, $q_2$). The two functions, $\phi_1(\cdot)$ and $\phi_2(\cdot)$ are increasing and $\phi_1(0) = \phi_2(0) = 0$.

   (a) Describe the three-dimensional production set associated with these two techniques. Assume free disposal.

   (b) Give sufficient conditions on $\phi_1(\cdot)$, $\phi_2(\cdot)$ for the production set to display additivity.

   (c) Suppose that input price are $w_1$ and $w_2$. Write the first order conditions for profit maximization and interpret. Under which conditions on $\phi_1(\cdot)$, $\phi_2(\cdot)$ will the necessary conditions sufficient?

   (d) Show that if $\phi_1(\cdot)$ and $\phi_2(\cdot)$ are strictly concave, then a cost-minimizing plan cannot involve the simultaneous use of the two techniques. Interpret the meaning of the concavity requirement, draw the isoquants in the two-dimensional space of input uses.

2. A price-taking firm producing a single product according to the technology $q = f(z_1, ..., z_{L-1})$ faces prices $p$ for its output and $w_1, ..., w_{L-1}$ for each of its inputs. Assume that $f(\cdot)$ is strictly concave and increasing, and that $\partial^2 f(z)/\partial z_l \partial z_k > 0$ for

all $l \neq k$. Show that for all $l = 1, ..., L-1$, the factor demand functions $z_l(p, w)$ satisfy $\partial z_l(p, w)/\partial p > 0$ and $\partial z_l(p, w)/\partial w_k > 0$ for all $k \neq l$.

3. A firm has a production function given by:

$$f(z_1, z_2, z_3, z_4) = \min\{2z_1 + z_2, z_3 + 2z_4\}.$$

   (a) What is the cost function for this technology?

   (b) What is the conditional demand function for factors 1 and 2 as a function of factor prices $(w_1, w_2)$ and output $y$?

4. Calculate the factor demands and the profit function in the following cases:

   (a) $f(z_1, z_2) = a_1 \ln z_1 + a_2 \ln z_2$

   (b) $f(z_1, z_2) = z_1^{\alpha_1} z_2^{\alpha_2}$

   (c) $f(z_1, z_2) = \min\{z_1, z_2\}^{\beta}$

   What restriction must be satisfied by $a_1$, $a_2$, $\alpha_1$, $\alpha_2$, $\beta$?

5. A firm cam produce good $q$ from factors $(z_1, z_2)$. Factor prices are $w \in \mathbb{R}^2_+$. The price of the output is $p \in \mathbb{R}_+$. The firm has a differentiable cost function $C(w_1, w_2, q)$. Restrict attention to interior solutions.

   (a) Show that a sufficient condition for this cost function to be strictly convex in $q$ is that the production function is strictly concave in $(z_1, z_2)$.

   (b) Assume for the remainder of the question that $C(w_1, w_2, q)$ is strictly convex in $q$. Obtain an expression of the profit-maximizing output response to a marginal increase in the output price.

   (c) Suppose now that factor 2 cannot be adjusted in the short run. As a result, the short-run cost function of the firm becomes

$$C_s(q|\bar{z}_2) = w_1 z_1 + w_2 \bar{z}_2$$

   where $z_1$ is chosen so that $f(z_1, \bar{z}_2) = q$.
   As in point (b), obtain an expression of the profit-maximizing output response to a marginal increase in the output price, when factor 2 cannot be adjusted.

(d) Using the results in (b) and (c), show that

$$\frac{\partial q^*(w,p)}{\partial p} \geq \frac{\partial q_S^*(w,p|\bar{z}_2)}{\partial p}$$

Explain the intuition behind this result.

6. Consider a firm with conditional factor demand functions of the form

$$\begin{aligned} z_1 &= 1 + 3w_1^{-1/2} w_2^a \\ z_2 &= 1 + b w_1^{1/2} w_2^c \end{aligned}$$

Output has been set equal to 1 for convenience. What are the values of the parameters $a$, $b$, and $c$ and why? Explain all the conditions you use.

Additional exercises that the students are encouraged to do:

1. A profit-maximizing firm buys its inputs at fixed prices and can sell its output in *one and only one* of the competitive markets. In market 1, the price is $p^-$ during the first day and $p^+$ during the second day ($p^+ > p^- > 0$). In market 2, the price is constant over time and equal to $\bar{p} = (p^+ + p^-)/2$, the average price of the other market. Assume that the firm cannot change the market after the first day and has to stay in the same market for two days. Further assume $\pi(p^-) > 0$, where $\pi(\cdot)$ is the profit function. Ignore any discounting between periods.

   (a) Which market would the firm prefer? To be precise, which market yields the highest profits over the two periods?

   (b) What a rational consumer (who needs to buy the firm's output and consume it everyday) would choose? which market he/she would prefer?

2. A firm has the following two factor production function $f(x_1, x_2) = x_1 + x_1 x_2$ , where $x_1$ and $x_2$ are amounts of inputs 1 and 2. The input prices are given by $w_1$ and $w_2$ and the quantity of output is denoted $y$.

   (a) Does this production function exhibit increasing, decreasing or constant returns to scale? Explain.

(b) Solve the firm's cost minimization problem, being certain to state for which values of $w_1$ and $w_2$ the conditional factor demands are non-negative.

(c) Using the solution of the previous point, express the firm's cost function, find marginal and average cost and explain why they are increasing or decreasing.

(d) Write down the competitive firm's profit maximization problem where it must choose levels of the two inputs.

(e) Solve for the profit maximizing factor demands by a competitive firm.

3. Joe is an "empire builder". That is, his goal is to produce and sell as much as possible. However, his stockholders impose the constraint that he looses no money. He operates using the production function $y = f(x)$, and faces parametric prices $p$ for his (single) output and (vector of) inputs. The production function is with positive marginal products.

   (a) Set up Joe's problem and state the first-order conditions.

   (b) Is the nonnegativity constraint on profit binding? Why or why not?

   (c) Interpret the Lagrangian multiplier. What is its sign?

   (d) Show that Joe's supply curve slopes upward.

   (e) Show that Joe's output is a decreasing function of all input prices.

4. MWG 5.C.7.

5. MWG 5.C.9.

6. MWG 5.C.10.

7. MWG 5.E.4.