Graduate Microeconomics I
Problem Set 3

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1. You are given the following information about a consumer's purchases. He consumes only two goods:

	Year 1		Year 2	
	Quantity	Price	Quantity	Price
Good 1	100	100	120	100
Good 2	100	100	?	80

Over which range of quantities of good 2 consumed in year 2 would you conclude:

   (a) That the consumer's consumption bundle in year 1 is revealed preferred to that in year 2 and WARP is satisfied?

   (b) That the consumer's consumption bundle in year 2 is revealed preferred to that in year 1 and WARP is satisfied?

   (c) That is behaviour is inconsistent, i.e., in contradiction with the weak axiom?

2. Ginger and Fred are married. Ginger takes all the decisions about how to spend their family income w and divide the purchases. She does this in such a way that she maximizes her utility function subject to the constraint that Fred's utility is at least equal to a given level u_f . This is what Fred would get after divorce. Let $x(p, w)$ denote the family demand vector while $x_f(p, w)$ and $x_g(p, w)$ are the consumption vectors of Fred and Ginger, respectively. Note that we have

$$x(p, w) = x_f(p, w) + x_g(p, w)$$

and recall that both x_f and x_g are chosen by Ginger. Show that $x(p, w)$ satisfies the Slutsky equation. (Hint: for the first step of the proof: Formulate the UMP and EMP for Ginger and show that the solution to one is a solution to the other.)

3. Suppose an economy composed by I consumers, and that the expenditure function of consumer i is of Gorman polar form:

$$e_i(p, u_i) = a_i(p) + u_i b(p) \quad \forall i$$

where $p = (p_1, ..., p_L)$ is the price vector. For consumer i,

   (a) determine the indirect utility function; and,

   (b) derive the demands for each good;

   (c) show that the demand function for any consumer can be generated by a "representative" consumer with indirect utility function given by

$$v(p, W) = A(p) + B(p)W$$

   with $W = \sum_i w_i$ and $A(p) = f(a_1, ..., a_I)$. Explain your results.

4. Suppose that there are I consumers and L commodities. Consumers differ only by their wealth levels w_i and by a taste parameter s_i, that we will call *family size*. Thus, denote the indirect utility function of consumer i by $v(p, w_i, s_i)$. The corresponding Walrasian demand function for consumer i is $x(p, w_i, s_i)$.

   (a) Fix $(s_1, ..., s_I)$. Show that if for any $(w_1, ..., w_I)$ aggregate demand can be written as a function of only p and the aggregate wealth $w = \sum_{i \in I} w_i$ and if every consumer's preference relationship $\succsim_i$ is homothetic, then all these preferences must be identical [and so $x(p, w_i, s_i)$ must be independent of s_i].

   (b) Give a sufficient condition for aggregate demand to depend only on aggregate wealth and on aggregate family size, i.e., $\sum_{i \in I} s_i$.

5. Suppose there are two consumers, Clark and Louise, with utility functions over two goods, 1 and 2, of $u_c(x_1, x_2) = x_1 + 4\sqrt{x_2}$ and $u_l(x_1, x_2) = 4\sqrt{x_1} + x_2$. The two consumers have identical wealth levels, $w_c = w_l = w/2$.

   (a) Calculate the individual demand functions and aggregate demand function.

   (b) Compute the individual Slutsky matrices $S_i(p, w/2)$, for $i = \{c, l\}$ and the aggregate Slutsky matrices.

(c) Show that the aggregate demand satisfies the Weak Axiom.

(d) Compute the matrix $C(p, w) = \sum_i S_i(p, w/2) - S(p, w)$ for prices $p_1 = p_2 = 1$. Show that it is positive semidefinite if $w > 16$ and that it is negative semidefinite if $w \in (8, 16)$.