Graduate Microeconomics I
Problem Set 2

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1. Consider a three-goods economy. Suppose that the prices of goods 2 and 3 always move in the same direction. Show then that goods 2 and 3 can be considered a single commodity, called a *composite commodity*. That is, show that the demand functions obtained from the original problem and the reduced problem (the latter using the composite commodity) are the same.

2. Consider an agent in a two-goods economy. For each of the preferences described below: (i) propose a utility function representing them and draw the indifference curves in the Cartesian space (x_1, x_2); (ii) calculate the marginal rate of substitution between these two goods and interpret; and (iii) examine whether the preferences are convex or not.

   (a) Good 1 has no effect on the welfare of Laura: she is indifferent between consuming it or not and she does not care about the quantity she uses. On the other hand, she always prefers having more good 2.

   (b) Martha prefers (x_1, x_2) to (y_1, y_2) if and only if $\min\{x_1, x_2\} \geq \min\{y_1, y_2\}$.

   (c) Maria is indifferent between the goods: she only cares about the total amount of good she has.

   (d) Cristina prefers (x_1, x_2) to (y_1, y_2) if and only if $x_1^2 + x_2^2 \geq y_1^2 + y_2^2$.

   (e) Sonia prefers (x_1, x_2) to (y_1, y_2) if and only if $x_1 x_2 \geq y_1 y_2$.

3. Consider the indirect utility function

$$v(p_1, p_2, w) = \frac{w}{p_1 + p_2}$$

   (a) What are the demand functions?

(b) What is the expenditure function?

(c) What is the direct utility function?

4. Let $(-\infty, \infty) \times \mathbb{R}_+^{L-1}$ be the consumption set and assume that preferences are striclty convex and quasilinear. Normalize $p_1 = 1$.

   (a) Show that the Walrasian demand functions for goods $2, ..., L$ are independent of wealth. What does this imply about the wealth effect of demand for good 1?

   (b) Argue that the indirect utility function can be written in the form $v(p, w) = w + \phi(p)$ for some function $\phi(\cdot)$.

   (c) Suppose $L = 2$, so we can write the utility function as $u(x_1, x_2) = x_1 + \gamma(x_2)$. Now, however, let the consumption set be $\mathbb{R}_+^2$ so that there is a nonnegative constraint on the consumption of the numeraire good, x_1. Fix prices p, and examine the consumer's Walrasian demand as wealth w varies. When is the nonnegativity constraint irrelevant?

   (d) Again, $L = 2$. Show that the Hicksian demand functions do not depend on a given level of utility, $\bar{u}$. What is the form of the expenditure function?

5. A consumer's expenditure function is given by:

$$e(p_1, p_2, u) = \alpha_1 p_1 + \alpha_2 p_2 + u p_1^{\beta_1} p_2^{\beta_2}$$

   (a) Derive (i) the Hicksian demand functions, (ii) the Marshallian demand functions, and, (iii) the indirect utility function.

   (b) State restrictions on the parameters α_1, α_2, β_1 and β_2, for the functions to be consistent with utility maximization.

6. Consider the utility function: $u(x_1, x_2) = 2x_1^{1/2} + 4x_2^{1/2}$.

   (a) Find that the demand functions for good 1 and 2 as they depends on prices and wealth;

   (b) find the compensated demand function $h(\cdot)$;

(c) find the expenditure function and verify that $h(p,u) = \nabla_p e(p,u)$;

(d) find the indirect utility function and verify Roy's identity.

7. Consider the indirect utility function:

$$v(p,w) = \frac{w}{\sqrt{p_1 p_2}}$$

Let $w = 10$ and consider a price change from $(p_1^0 = 2, p_2^0 = 2)$ to $(p_1^1 = 1, p_2^1 = 1)$.

(a) Calculate the equivalent and compensating variations EV and CV .

(b) Compare EV and CV. Would you obtain the same type of comparison for any type of preferences? If not, explain which factors explain the comparison between these two measures.

8. Ellsworth's utility function is $U(x,y) = \min\{x,y\}$. Ellsworth has 150 and the price of x and the price of y are both 1. Ellsworth's boss is thinking of sending him to another town where the price of x is 1 and the price of y is 2. The boss offers no raise in pay. Ellsworth, who understands compensating and equivalent variations perfectly, complains bitterly. He says that although he does not mind moving for its own sake and the new town is just as pleasant as the old, having to move is as bad as a cut in pay of A. He also says that he would not mind moving if, when he moved he got a raise of B. What are A and B equal to?

Additional exercises that are encouraged to the students do:

1. Let $x_l(p,w)$ denote the Marshallian demand for good $l = 1, ..., L$, where p stands for the price vector and w consumer's wealth.

(a) Show that for homothetic preferences we have

$$\frac{\partial x_l}{\partial p_k} = \frac{\partial x_k}{\partial p_l} \quad \forall l, k = 1, ..., L$$

i.e., the cross price effects are symmetric.

(b) Is this true across all utility functions? Justify your answer.

2. Consider the normalized price vector,

$$q = \frac{1}{w} p \quad \in \mathbb{R}^L,$$

where p is the price vector, while w is income. Write the indirect utility function as a function of q only, i.e., $v(p, w) = v^*(q)$. Preferences are said to display indirect additivity if

$$v^*(q) = f\left(\sum_{l=1}^{L} v_l(q_l)\right)$$

where $f(\cdot)$ is an increasing function ($f' > 0$). Show that indirect additivity implies that for all distinct i, j, k, the price elasticity of good i with respect to the price of good k is the same as the price elasticity of good j with respect to the price of good k.

3. Given a utility function

$$U(x_1, x_2) = \frac{x_1^\rho}{\rho} + \frac{x_2^\rho}{\rho}$$

derive:

   (a) the uncompensated demands for x_1 and x_2,

   (b) the indirect utility function,

   (c) expenditure function, and

   (d) the Hicksian demand functions for x_1 and x_2

4. MWG 3.D.5.

5. MWG 3.D.6.

6. MWG 3.D.7.

7. MWG 3.E.6.

8. MWG 3.E.7.

9. MWG 3.G.3.

10. MWG 3.I.3.

11. MWG 3.I.8.