Graduate Microeconomics I
Problem Set 1

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1. Show that if $u(\cdot)$ is a continuous utility function representing $\succsim$, then $\succsim$ is continuous.

2. Show the following statements:

   (a) If $\succsim$ is strongly monotone, then it is monotone;

   (b) If $\succsim$ is monotone, then it is locally non-satiated.

3. Draw a convex preference relation that is locally non-satiated but is not monotone.

4. Show that if $f : \mathbb{R} \to \mathbb{R}$ is a strictly increasing function and $u : X \to \mathbb{R}$ is a utility function representing preference relation $\succsim$, then the function $v : X \to \mathbb{R}$ defined by $v(x) = f(u(x))$ is also a utility function representing relation $\succsim$.

5. Consider a rational preference $\succsim$. Show that if $u(x) = u(y)$ implies $x \sim y$ and if $u(x) > u(y)$ implies $x \succ y$, then $u(\cdot)$ is a utility function representing $\succsim$.

6. Establish the following results:

   (a) A preference relation which can be represented by a homogeneous (of any degree) function is homothetic.

      i. Are lexicographic preferences homothetic?

   (b) A continuous and monotone preference relation is homothetic if and only if it admits a utility function $u(x)$ that is homogeneous of degree one, i.e., $u(\alpha x) = \alpha u(x)$ for all $\alpha > 0$.

   (c) A continuous preference relation on $(-\infty, \infty) \times \mathbb{R}^{L-1}_{+}$ is quasilinear with respect to the first commodity if and only if it admits a utility function $u(x) = x_1 + \phi(x_2, ..., x_L)$.

7. Suppose that in a two-commodity world the consumer's preferences are represented by a Constant Elasticity of Substitution (CES) utility function, $u(x, y) = [\alpha_x x^\rho + \alpha_y y^\rho]^{1/\rho}$. Show that:

   (a) if $\rho = 1$, then the indifference curves become linear;

   (b) if $\rho \to 0$, then preferences are represented by a Cobb-Douglas utility function, $u(x, y) = x^{\alpha_x} y^{\alpha_y}$; and,

   (c) if $\rho \to -\infty$, then the indifference curves become "right-angled"; in other words, preferences are represented in the limit by a Leontief utility function, $u(x, y) = \min\{x, y\}$.