Graduate Microeconomics I

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Problem set 1

Preferences and Decision theory (1)

1) (a) Show that if $\succsim$ is strongly monotone, then it is monotone. (b) Show that if $\succsim$ is monotone, then it is locally non-satiated. (c) Draw a convex preference relation that is locally non-satiated but is not monotone.

2) Show that if $f : \mathbb{R} \to \mathbb{R}$ is a strictly increasing function and $u : X \to \mathbb{R}$ is a utility function representing the preference relation $\succsim$, then the function $v : X \to \mathbb{R}$ defined by $v(x) = f(u(x))$ is also a utility function representing relation $\succsim$.

3) Consider a rational preference relation $\succsim$. Show that if $u(x) = u(y)$ implies $x \sim y$ and $u(x) > u(y)$ implies $x \succ y$, then u is a utility function representing $\succsim$.

4) Consider an agent in a two-goods economy. For each of the preferences described below: (i) propose a utility function representing them and draw the indifference curves in the Cartesian space (x_1, x_2); (ii) calculate the marginal rate of substitution between these two goods and interpret; and (iii) examine whether the preferences are convex or not.

(a) Good 1 has no effect on the welfare of Laura: she is indifferent between consuming it or not and she does not care about the quantity she uses. On the other hand, she always prefers having more good 2.

(b) Martha prefers (x_1, x_2) to (y_1, y_2) if and only if $\min\{x_1, x_2\} \geq \min\{y_1, y_2\}$.

(c) Maria is indifferent between the goods: she only cares about the total amount of good she has.

(d) Cristina prefers (x_1, x_2) to (y_1, y_2) if and only if $x_1^2 + x_2^2 \geq y_1^2 + y_2^2$.

(e) Sonia prefers (x_1, x_2) to (y_1, y_2) if and only if $x_1 x_2 \geq y_1 y_2$.

5) Establish the following results:

(a) A preference relation which can be represented by a homogeneous (of any degree) function is homothetic.

i. Are lexicographic preferences homothetic?

(b) A continuous and monotone preference relation is homothetic if and only if it admits a utility function $u(x)$ that is homogeneous of degree one, i.e., $u(\alpha x) = \alpha u(x)$ for all $\alpha > 0$.

(c) A continuous preference relation on $(-\infty, \infty) \times \mathbb{R}_+^{L-1}$ is quasilinear with respect to the first commodity if and only if it admits a utility function $u(x) = x_1 + \phi(x_2, ..., x_L)$.

6) Let $(-\infty, \infty) \times \mathbb{R}^{L-1}$ be the consumption set and assume that preferences are strictly convex and quasilinear. Normalize $p_1 = 1$.

(a) Show that the Walrasian demand functions for good 2,...,L are independent on wealth. What does this imply about the wealth effect of demand for good 1?

(b) Argue that the indirect utility function can be written in the form $v(p, \omega) = w + \phi(p)$ for some function $\phi(\cdot)$.

(c) Suppose $L = 2$, so we can write the utility function as $u(x_1, x_2) = x_1 + \gamma(x_2)$. Now, however, let the consumption set be $\mathbb{R}_+^2$ so that there is a non-negative constraint on the consumption of the numeraire good, x_1. Fix prices p, and examine the consumer's Walrasian demand as wealth ω varies. When is the nonnegativity constraint irrelevant?

(d) Again, $L = 2$. Show that the Hicksian demand functions for good 2 do not depend on a given level of utility $\bar{u}$. What is the form of the expenditure function?

7) Consider the indirect utility function

$$v(p_1, p_2, w) = \frac{\omega}{p_1 + p_2}$$

(a) What are the demand functions?

(b) What is the expenditure function?

(c) What is the direct utility function?