Lecture 7: Non-equilibrium Strategic Thinking

Till now: Game theoretic rationality with non-egoistic preferences

Now: models of boundedly rational behavior in games

Lecture 8: Learning models

This lecture: non-equilibrium behavior

models behavior in "new", unfamiliar strategic situations before players can learn from experience

1. Quantal Response (McKelvey and Palfrey 1995, Goeree, Holt, and Palfrey 2008)

Basic features

Player choose "noisy" best response

In equilibrium the likelihood of a strategy is increasing its expected payoff, taking the noisy strategies of the other players into account

n player normal form game G (or normal form representation of an extensive form game)

S_i: finite set of pure strategies of player i, $s_i \in S_i$

S: set of pure strategy profiles, $s \in S$

s_{-i} strategies of all players but i

each player i endowed with payoff-function:

$$\pi_i : S \to \mathbb{R}$$

mixed strategy of player i, σ_i: probability distribution over S_i

$\sigma_i \in \Sigma_i$

$\sigma = (\sigma_1, \sigma_2, ...\sigma_N)$, $\sigma \in \Sigma = \prod_{i \in N} \Sigma_i$

$i's$ strategic behavior can be summarized by a function P_i: $\mathbb{R}^{|S_i|} \rightarrow \Sigma_i$

$P_{ij}(u_i)$: For a vector $u = (u_{i1}, u_{i2}, ...u_{i|S_i|})$, $P_{ij}(u_i)$ denotes the probability that player i chooses pure strategy j, if his payoff of choosing his strategy 1 is u_{i1}, of choosing his strategy 2 is u_{i2}, etc.

Definition: P_i is a quantal-response function, if the following conditions holds:

Continuity: $P_{ij}(u_i)$ is continuously differentiable at all u_i

Interiority: $P_{ij}(u_i) > 0$ for all u_i.

Responisveness: $\frac{\partial P_{ij}(u_i)}{\partial u_{ij}} > 0$ for all j at all u_i.

Monotonicity: $u_{ij} > u_{ik} \iff P_{ij}(u_i) > P_{ik}(u_i)$ for all $j, k \in S_i$

All conditions exclude that P_i is equavalent to a best response correspondence, but conditions 3 and 4 keep the flavor of a best response $\iff P_i$ models a smoothened, non-precise version of best response.

Definition: Take a normal form game G and let $P = (P_1, P_2 ... P_N)$ be a profile of quantal response functions. A mixed strategy profile $\sigma^* \in \Sigma$ is a quantal response equilibrium (QRE) iff $\sigma^* = P(\sigma^*)$.

Theorem: For any game G and any profile of quantal response functions P there exists a QRE.

Theorem: Let P^n be a sequence of profiles of quantal response functions that converges to a profile equivalent to the best responce correspondances. The resulting sequence of QREs converges to a Nash equilibrium.

QRE can explain many experimental results, but:
QRE depends on P which is of course not unique - too many degrees of freedom

2. Level-k-Models

2.1. Experimental evidence

Guessing game ("beauty contest") (Nagel 1995, Ho et al. 1998):

n players choose simultaneously a number between α and β; $\alpha \geq 0$, $\beta > \alpha$

The person whose number is closest to x times the average number wins. Ties broken randomly

Unique Nash equilibrium:
$x < 1$: all players choose α.
$x > 1$: all players choose β.

This equilibrium also results from iterated elimination of strictly dominated strategies

Experimental results in early rounds:

Most players choose numbers larger than zero

Spikes at $\frac{\alpha+\beta}{2}x^k$, $k = 1, 2$, and 3.

Inconsistent with quantal response

2.2. The model (Nagel 1995, Stahl and Wilson 1995)

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S_i: finite set of pure strategies of player i, $s_i \in S_i$

S: set of pure strategy profiles, $s \in S$

s_{-i} strategies of all players but i

each player i endowed with payoff-function:

$$\pi_i : S \longrightarrow \mathbb{R}$$

Players are heterogenous and charaterized by their sophistication level.

A L(evel)0 player i makes a random choice according to some probability distribution over S_i (for many applications equal distribution over feasible strategies assumed). Such $L0$ players do not actually exist, but they are needed to anchor the belief of all other players.

$L1$, $L2$, $L3$...players play best response, but differ in terms of beliefs about what the other players do.

A $L1$ player i chooses a strategy s_i that maximizes his payoff under the believe that all other players are $L0$.

A $L2$ player i chooses a strategy s_i that maximizes his payoff under the believe that all other players are $L1$.

A $L3$ player i chooses a strategy s_i that maximizes his payoff under the believe that all other players are $L2$.

etc.

Applied to the guessing game:

$L0$: random choice with equal distribution over numbers between α and β. choice of strategy is random

$L1$ players believe that othere players are $L0 \Rightarrow$ average is $\frac{\alpha+\beta}{2} \Rightarrow$ best response is $\frac{\alpha+\beta}{2}x$

$L2$ players believe that othere players are $L1 \Rightarrow$ average is $\frac{\alpha+\beta}{2}x \Rightarrow$ best response is $\frac{\alpha+\beta}{2}x^2$

$L3$ players believe that othere players are $L2 \Rightarrow$ average is $\frac{\alpha+\beta}{2}x^2 \Rightarrow$ best response is $\frac{\alpha+\beta}{2}x^3$

Level-k models predict well actual behavior in many economically important experimental games, e.g. auctions

Variant of level-k model: Cognitive Hierarchy Model (Camerer at al 2004):

Lk players do not play best response against belief that only $L(k-1)$ players exist, but against a belief given by a distribution over all $L(k-m)$ players with $m = 1, 2...k$.

Believed distribution of $L(k-m)$ players is such that the relative frequency of all types are correct - A players does not realize that other players might of the same or higher level than himself, but he correctly predicts the ratio between $L(k-m)$ and $L(k-n)$ players $(m, n \in \{1...k\})$

Shortcomings of level-k models:

Cannot capture learning well - "short term" predictions

In many games, predictions depend on assumption about $L(0)$, and on size of $L(1)$ players, etc.

Unclear, whether predictions are better than with QRE (Breitmoser 2012)